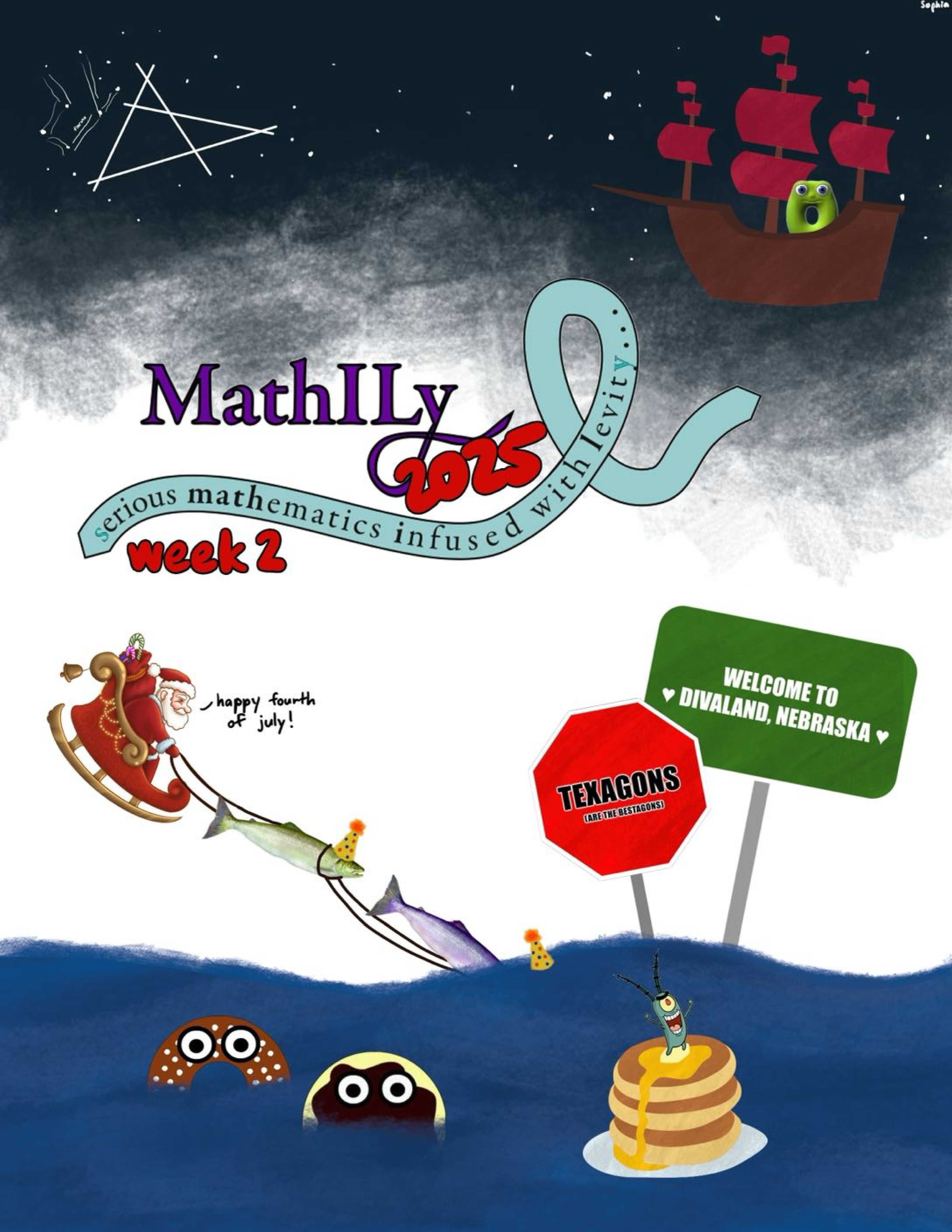

Sophia
MathILy 2025
serious mathematics infused with levity...
week 2
happy fourth of july!
WELCOME TO ♥ DIVALAND, NEBRASKA ♥
TEXAGONS
(ARE THE BESTAGONS)

the MathILy Record of Mathematics (RoM)
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Welcome…

…to the 2nd issue of the Record of Mathematics, MathILy 2025! As Root comes to a close, despite our sadness and anticipation, we've continued learning and developing our skills. In sarah-marie's root, eggs spawned salmon spaces with fancy fishened fishboxes. The students in Brian's root swirled concoction kits and powered Titanium Titans Party Parrots invaded Hannah's root while they contemplated Secret Santa selections. Every root gained new insights into mathematical structures and the connections between them, which will aid them in the chaotic week to come. For Daily Gather, we dug up fossils, connected Texagons, and discovered the secret of isometries – it's not about the action, it's about the correspondence.

Read more in Root Classes!

In this issue…

2.1 sarah-marie root BY Andrew Chai, Brandon Furman

Fishats and THEC sarah-marie told us that fish have very particular hat preferences. Two schools of fish cannot show up to a party wearing the same kind of hat, and no hat can go unworn. We also learned that David is a kind of fish, which is distinct from a suba diver. Also, a diagonal triangle is a kind of hat. In a novel-worthy proof, Andrew demonstrated that we can make a successful fish hat (fishat) if and only if every subset of fish like at least as many hats as in the subset of fish and vice versa for hats.

Aquariums Next, sarah-marie showed us an aquarium of earth animals. The alien salmon liberated the aquarium by breaking individual glass walls to release the animals in each exhibit. We realized that there were two hidden school districts in this liberated aquarium: the architecture of the aquarium could be seen as a school district with unbroken walls as lanes and columns as schools, and the liberation path could be seen as a school district with broken walls as lanes and exhibits as schools.

Counting Frogs Vera described awesome frogs to us. Some of these frogs breathed fire, some could fly, and some could play violin (some could do all three at the same time). We proved a claim about how to calculate the total amount of frogs using intersections.

Hilbert's Hotel sarah-marie read us a story about the Smiths' vacation to the Hilbert Hotel. The hotel was fully booked, but there was still room for the Smiths. There was even space to accommodate an infinite amount of coaches each holding an infinite amount of people. One visitor even got an infinite amount of pudding. However, there were not enough rooms to accommodate every club since the Absentees Club had no meeting room, demonstrating that $|P(\mathbb{N})| > |\mathbb{N}|$.

ProbaBrandon We delved into the ideas of probaBrandon with Brandon variables and their expected value's. One of our two definitions of expected value put Brandon in jail. Brandon turned out to be his own jailer and his own savior when he proved that the two definitions of expected value were exactly the same, meaning Brandon's jail status was always irrelevant.

Smooshin' F's We put F's in the chat for our misunderstandings of linear mappings when we smooshed our F's, fishbox style. This demo was a fun and unique way of realizing that the F has always been inside of us, waiting to be smooshed.

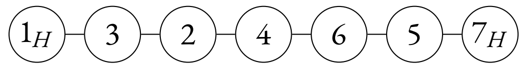
New Uses of Fishboxes This week, we found more ways to use fishboxes. We proved that fishboxes can represent useful moves to transform other fishboxes, and that they can rotate salmon points as well. Also, every linear mapping can be represented as a fishbox and vice versa. Through painful subscript manipulations, Brian proved that fishbox multiplication is associative.

Shining Lights Through Polyhedra David gave us flashlights to shine at polyhedra. We conjectured positions to place the flashlights relative to the polyhedra such that edges didn't cross in their shadows. Next, we noticed that such shadows resembled aquariums, and we counted the total angles in of them in two different ways to show a result about the number of vertices, edges, and faces of all polyhedra.

Rotation Groupers One day, sarah-marie told us to consider the symmetries of regular polygons. Given a polygon with n sides, we have n ways to rotate the polygon about its center and n ways to flip it about

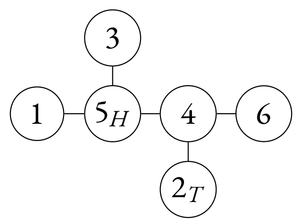
3.2 Josh—Spooky Scary Fossils BY Thomas Amdur

To understand fossils, we must first understand spines, which can be depicted as a line segment, containing numbered joints and bones connecting them:



In this example, the head must be the leftmost joint and the tail must be the leftmost. The class argues and proves that there are $n!$ unique spines with n joints, each corresponding to a distinct permutation of joints. A mysterious "Grant" suggests that these permutations be bijected with bijections $f : \{1, \ldots, n\} \to \{1, \ldots, n\}$. Ben suggests a possible bijection - a spine can be read starting from the head, which is $f(1)$, to the tail, which is $f(n)$.

Josh then draws a fossil slightly more complicated than shown:



which is more versatile than a spine because joints, bones, heads, and tails can be placed anywhere as long as no "ribcages"—or loops—are formed, and one (disjoint) fossil is considered at a time. Josh asks how many unique fossils can be formed with n joints. Aidan and Spark convince the class that with $n = 1, 2, 3$ joints, there are 1, 4, 27 fossils, respectively.

After some more thinking, Miles comes up with a bijection construction to a function $f : \{1, \ldots, n\} \to f : \{1, \ldots, n\}$. He explains that each fossil has a unique spine from its head to its tail, and the numbers on the spine can be SORTED from head to tail and mapped to the actual index of the joint they end up on. Every other number maps to an adjacent number closer to the spine. For the backwards direction, given any function f, identifying all "cycles" occuring from repeatedly applying f tells us which numbers need to be in the spine. Then one unique way can be found to index one and create the "branches" of the fossil outside the spine. To exemplify this bijection, the six-joint fossil previously drawn would give us the following:

x	f(x)
1	5
2	5
3	5
4	4
5	2
6	4

Since any function f is allowed, we have n^n unique fossils with n joints, and n^{n-2} if we decide not to consider heads and tails.

3.3 David—Doodling in Class BY Spark Shulman

On Wednesday, David taught us the better way to doodle: writing sequences of fractions in your notebook, and multiplying an integer by them. Obviously.

He began with this demo-doodle:

$$\frac{3}{5}, \frac{4}{7}, \frac{25}{2}, \frac{11}{13}$$

To use a doodle, we (apparently) have to follow certain steps:

1. Start with a number (say 4)
2. Go left to right, looking for a fraction to multiply by, so that you get a whole number answer
3. Multiply and get a new number
4. Repeat with the new number
5. End when you can't go any more

We start the demo-doodle, using 4, and end up "dying" after 6 moves, having reached the number 81. David tells us that these doodles are actually functions, where we code our inputs in prime factorizations.
A *unary* function, like $f(x) = y$, takes input 2^x and outputs 3^y.
A *binary* function, like $f(x, y) = z$, takes input $2^x 3^y$ and outputs 5^z.

We explored more doodle-functions, including David's unary doodle from Spanish class:

$$\frac{7}{11 \cdot 2}, \frac{11}{7 \cdot 2}, \frac{7 \cdot 3^k}{5}, \frac{5}{2}, \frac{1}{7}, \frac{1}{11}$$

...which returns k, and his binary doodle from Econ class:

$$\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}$$

...which returns $x \cdot y$.

We also figure out how to compose two doodle-functions, shifting all the primes around to other primes so that they don't interfere.

Finally, when we think we can't be any more impressed by the capabilities of doodles, David reveals something unbelievable. Not only can he add and multiply with doodles, and compose them, but a doodle is actually as versatile as a computer!

3.4 Math Movies 2 BY Eric Barajas

Mobius Transforms Revealed We know that dilations, translations, and rotations are some pretty conventional transformations. Let's project it on a sphere and let it determine the type of transformation we want! We can now invert space to our will! Inversions of graphs with this type of transformation are possible now.