
VOTE
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Chaos Edition

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Welcome...

In the chaotic third week of MathILy, we split up into 26 different classes. The topics discussed spanned from various types of numbers (cool numbers and peals, just to name a few) to applied math (such as voting theory, combinatorial optimization, and game theory), and, of course, "why bubble no cube??" Brian and Other Brian competed, messing with each other's notes in Linear Counting and Polynomial Counting until they eventually came to an agreement at the end of the week (they've both combined to become just "Brian" again). Through the daily gathers, we learned how polynomials are like integers, but better and how to correctly hold a pizza. Our "mystery" guest also taught us how to effectively drop eggs and soup in a harsh economy, and Berit intrigued us with her seemingly innocent problem. Lastly, we discussed the risk of buying weather-dependent cows and how to quantify said risk. Next week, look forward to branching out and either exploring d-dimensional canopies or having an existential crisis (why is there anything at all?).

Read more in Week of Chaos Summaries!

In this issue...

it was a suit/healthy bag/imperial hood over the vibrophone/salad/universe of solutions to the recurrence. Next, we gave upper bounds on the degree of the recurrence formed by manipulating two recursive sequences in various ways, such as adding them, multiplying them, etc. Finally, we found a way to instantly prove or disprove many Fibonacci identities, using the fact that each one could be rearranged to form a recurrence which is a manipulation of many others, so it always holds if it holds for the first "few" cases. Now, all that remains is to check the 32768 base cases!

2.8 Information Theory with Katie BY Matt and Yuhuan

More Human than Humachine(?) We humans have language, but all languages are pretty inefficient. All of these vowels and symbols take up a whole lot of space and time. Some would argue ... *too much time.* Sure, we can delete letters here and there, and maybe get rid of a few spaces, but that just isn't all that fast. Thousands of years of human evolution simply isn't good enough; we must push past our human barriers. So, how can we become more like the machines we're destined to be, and lose our humanity along the way? Simple! Computers are pretty fast, so we'll just learn how to turn human language into computer language. Surely there won't be any consequences.

Wndrfl Wys to Cmprs Infrmtn Compress our language (and any language, really) into a series of bits. We can't just choose any correspondence from characters to codewords (made of bits), though. Instead, we'll want to design a language to be readable, unique, and most importantly **speedy**. Call a language that follows these constraints an optimal language. Then, how can we create an optimal correspondence between characters and strings? Surprisingly, we can construct a binary tree, with each of the codewords as leaves. With this condition, we may actually guarantee that a message can't be misinterpreted by cutting it off early. How should we actually design this binary tree, though? If we want to be fast, the most common characters should have the shortest codewords. Then, we may either build a tree from the top down (starting with high probability) or from the ground up (starting with low probability).

Really Squeezing Out the Information As the binary trees grew, we were left with one fundamental problem: how much information can one language carry (you know, because we're in information theory)? The least probable codewords should carry the most information, while the most probable carried the least. With that, we found an upper bound on the information a language could actually carry. Finally, as the week came to a close, we came up with the perfect language: *love*. Just kidding, it's still binary.

2.9 To p and beyond! with Frank BY Adam and Cece

In this class, we figured out how to describe and use p-eal numbers to do lots of cool stuff. First, however, we had to convince Frank that real numbers even exist in the first place. We defined convergent and "Walter" (stronger than convergent) sequences and used them to define real numbers. This used the absolute value function a lot, so Frank showed us that it was a "norm"—a function like absolute value that satisfies three properties—and asked us to find a new norm function based on how much of a given prime p was in a number. After a few failed experiments, we settled on $norp(p^n * \frac{a}{b}) = p^{-n}$ for some p. This allowed us to define p-eal as p-alter sequences of numbers that extend infinitely in the increasingly *positive* exponent direction rather than the traditional negative direction, making for some weird and initially counter-intuitive representations of numbers. After calculating some representations for rational numbers, we showed that all rational representations must eventually repeat, and moved on to irrationals. Radicals were pretty tricky (although we can perfectly easily represent $\sqrt{-1}$ in most p-eals!), and we found a few conditions that make our algorithm fail on some irrational numbers. Finally, we tried to represent polynomials in the p-eals, and found some conditions for a polynomial to be representable involving some unexpectedly sneaky derivatives.

2.10 Linear Counting with Other Brian BY Sophie

Brian wants to know how many guests he can invite to his fashionable party. However, each year, there are strict fashion taboos to be observed. In 2024, one cannot wear an even number of accessories or share an odd number of accessories with another guest. And, of course, wearing the same outfit as someone else is absolutely not allowed. For n fashionable accessories, we first constructed a lower bound. Then, carrot vibes in $\mathbb{Z}_2^n$ and their dot products were used to figure out the maximum number of guests possible.

After that, we planned next year's party (according to Brian's contacts at Vanity Fair): don't wear an odd number, don't share an even number of accessories. We adopted many of the same strategies but eventually had to split into cases of whether there were an odd number or even number of accessories.

Brian has heard the "in" trends of 2030 from Vogue (pssst... Anna Wintour is *our* vogue connection.) Now, we can only wear an even number and share an even number. We constructed salads of outfit vibes in order to use Dim Sum to prove what we conjectured...

What a fashionable party! And how fashionable those geometers, especially! How can Brian impress them? Well, by answering the question of how many points one can put in $\mathbb{R}^d$ such that every pair of points is the same distance away or the same two distances away. While pondering this, BRIAN left a little hint: what polynomial would go through all but one of these points? Using linear algebra, we were again able to find minimum and maximum bounds for these points.

2.11 Origametry with Tom BY Michael C

We learned about the wonderful mathematical properties of paper folding! We began by experimenting with folding equilateral triangles and other regular polygons (can you fold a pentagon from a square sheet of paper? A hexagon? A heptagon?). To formalize our constructions, we came up with a set of legal origami moves, analogous to the rules of classic straightedge and compass constructions. We found that origami allows for unique moves such as folding a point P_1 onto a line ℓ such that the crease goes through another point P_2. These properties of folding allowed us to do crazy things such as trisecting angles [gasp!] and finding common tangent lines between parabolas. Later, we delved into modular origami (folding lots of little units to form something big), using PHiZZ units to make polyhedra and figuring out which kinds could be constructed based on the number of pentagonal and hexagonal faces and leveraging knowledge from our study of sparsuckerpodes/plamp cities/minty molds in root. We ended by looking at the rules governing creases that converge at a point, and concluded that the number of "mountain" creases and the number of "valley" creases must differ by two in order for the paper to fold flat.

2.12 Euclid Shmeuclid with Nate BY Adam and Narnia

Euclid's book *Elements* introduced the idea of building up a geometric system with proofs from a set of five main axioms – that there exists a unique line between any 2 points, that line segments can be extended, a circle exists given any center and radius, all right angles are the same, and that there exists a unique parallel line through a point not on a line. But what if we decided to break axiom 5?

Let's imagine a sphere, where lines are great circles connecting two points. Although we were able to show that all of the first 4 postulates hold (resolving any issues by considering points and their antipodal the $x_1^{|A_1|-1}x_2^{|A_2|-1}\cdots x_n^{|A_n|-1}$ coefficient of a polynomial given its outputs on $A_1 \times A_2 \times \cdots \times A_n$. This led to the Kyle-jecture, giving us a way to show there exist nonzero outputs of a polynomial given some special coefficient is nonzero. Using this fact and carefully constructed polynomials, we solved problems about choosing 100 numbers around a circle, planes in space, and even sparsuckerpodes!

2.25 Which shape best shape? with Nate BY Gabe

Why Does Mirrors? Point. Point. River. Flip point over river. Make line. Flip back. Mirror. Angles same.

Why Slime Mold Know Angle? Trangle three point. Distance mimize. 120 degree angle. But what if cannot? Make big angle. So slime mold minty mold. Sucker three arm. 120 degree.

Why Bubble No Cube? n-gon. Fix ara. Mimize periter. Or fix periter. Mamize ara. We suck. Nate give easier problem. n points. $n-1$ lengths fixed. Make length so mamize ara. Lotta right angles. Extend to semicircle becuz every angle right. So split into two piece. Both semicircle. Boom mamize ara. But mamize ara = mimize periter. By scale. Then other surface. On square, sometimes quarter circle sometimes line. On horn, decrease periter *ad infinitum*. On cylinder, sometime two circle sometime one. On sphere, hard to prove because Nate mean but always circle. Torus sometime circle sometime lines. BUT WHY BUBBLE NO CUBE. Because planes split two. Reflection. So symmetry everywhere. Whoa. Sphere.

Which Shape Best Shape? Democracy. Not circle. Not hexagon. Not oblong spheroid. Why? Because ♩♪♩*I'm in love with the* ***shape of you...*** ♩♪♩

2.26 Why We Can't Have Nice Things with Ian BY Adriana and Darren

The dining hall has lost almost all their money, and now they can only afford to supply one food item for the rest of MathILy! Luckily, Erdman decides to have us vote for which food we would like to have. First, we came up with a couple of voting systems to use to determine our winner.

These voting systems were: Leo Dictatorship, in which the outcome is just whatever Leo's ballot says; Points, in which we sum up the place values of the voter ranks for each of the candidates and whoever has the lowest wins; Random, in which we pick the outcome randomly; Most Firsts, in which the candidate with the most 1st place votes wins; Delete Last, in which the candidate with the least 1st place votes gets removed from all of the ballots and everyone moves up accordingly, repeat; and lastly, Stone Always Wins, in which the dining hall decides to serve only rocks.

We then came up with properties we wanted our voting system to satisfy. Equality, No Stoners, Unanimity, Relativity, Winners Win, Losers Lose, and Majority. We looked at our systems to determine whether or not they satisfied our properties. Based off this data, we made conjectures relating to our properties. For example, Michael C proved that a voting system can't satisfy both Majority and Relativity.

Lastly, we proved that the only system with Winners Win, Unanimity, and Relativity is a Leo dictatorship by creating oligarchies in which if all of the voters within the oligarchy vote one candidate above another, in the final ballot, that candidate will be placed above the other. Then, we showed that if a voting system has Winners Win, Unanimity, and Relativity, all oligarchies have a smaller oligarchy within them, going all the way down to a 1 person oligarchy- a Leo Dictatorship.