
MATHILY
LEVEL 2

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Welcome...

In the second week of MathILy, our roots have reached their ends. sarah-marie's root took revenge on the dastardly tree octopus, who inked the skunk's lair and promptly stole its snacks. Spain has been conquered through gerrymandering, numbats have gotten their matching socks, and we even had time to enjoy some Dim Sum, sandwiches, and donuts. Nate's root has metamorphosed and is now extremely fashionable, bearing outstanding vibes, megaphones, and numerous tricks up their sleeves. Brian's root continued to cook up scrumptious (albeit moldy) cuisine, specializing in freshly farmed carrots, spicy stews, various kinds of salad, and of course, egregious amounts of cheese. We welcomed several guests this week, who bestowed us with riddles, career advice, risky tic-tac-toe, and interpersonal quilt-making skills. We wrapped up root by building polyhedra and introspecting. That's all for this week! Prepare for the chaos...

In this issue...

3 Daily Gathers

3.1 Peculiar Polebug Propinquity BY Leo Tsai

woah it's a polebug party Ian started the Daily Gather with some fun facts about polebugs! Polebugs live on poles. Each day, a polebug emerges from its shell, and leaves behind three descendants: one going down-then-left, down, and down-then-right, with the left and right descendants moving laterally until they come across another polebug.

Using our newfound knowledge of polebugs, we then counted the number of polebugs at each point, cutting up the pole to form a plane.

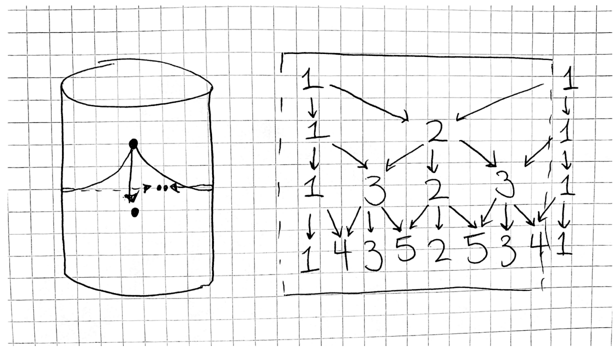

Figure 1: polebug <3

We then observationized observations:

- The first number of a row is always a 1.
- The number of clumps of polebugs doubles each time.
- The max number in a clump on day n is F_{n+1}.

Then, we went in groups to try and prove that the maximum number on day n is F_{n+1}. Jacob used induction on row n to prove that the Fibbonacci numbers do appear. Following this discussion, this ultimately led to the claim that

$$\max(\text{Row } n) \leq \max(\text{Row } n-1) + \max(\text{Row } n-2).$$

After Ian defined P_n = nth number in the sequence obtained from reading the pole, we realized that P_n could be derived from

$$P_1 = 1, \qquad P_{2m} = P_m, \qquad P_{2m+1} = P_{m+1}.$$

Then, we looked at $\frac{P_n}{P_{n+1}}$. A wild bold claim appeared! A lot of people claimed that this sequence (1) hits every positive rational (2) exactly once, and (3) in lowest terms.

The rest of the Daily Gather was for us to try and prove these three conditions. Stephen and Kyle ended up proving claims (1) and (2) before the Daily Gather ended.

3.2 Three Puzzles and AoC BY Benjamin T

(Guest: Tarik Agouab, Haverford)

We started out with an axiom called the Axiom of Choice (AoC) which states that if you have some a set of sets (potentially infinitely and uncountably many of them) and each is nonempty, you can always find a way to pick exactly one item from each set. This seems obvious, right? Just choose the smallest one or something. It really is quite simple for finite sets, but the fact that this works for infinite sets gives some weird results, even though it is taken as true.

We then continued to a series of riddles. For the first one, you have some finite number of people standing in a line with hats on that are either black or white. They can only see the hats of those in front of them in line and scary prison-keeper Nate is making them guess what hat color they have on. As it turns out, it is possible to have only one person potentially get their hat color wrong and for the rest to escape.

We then turned to the same problem, but with infinitely many people standing in the line. This time it is enough just to have a finite number of people fail, not just one. Though this might seem impossible or at least nonsensical, Tarik introduced how we could use the axiom of choice to create equivalences between hat orderings that are "all but finitely equal" and Nat showed us how we could use that to make only a finite number of people get their color wrong.

For the last riddle, we have a finite number of people with perfect memory and logic each go into their separate rooms, each with an infinite row of boxes each with a real number in it. Each person can open some of the boxes, but not all of them, and then must guess the number in that box. Surely this is impossible since each box has a number that is independent of all of the other numbers in boxes in that room, but is it?

3.3 Math Movies! BY Matt F

Rhythmetic Wacky funky beats, but enumerated! Lots of additions, a little bit of subtraction, and a whole diamond of fun. $0 = 0$ in a lot of fun ways. We can also count to $\{1, 2, 3, 4, 5\}$ in about 100 different ways, apparently.

Dirac's Belt Trick, Topology, and Spin $\frac{1}{2}$ Particles Twist a belt! Twist it, and we can't untwist it. Twist it too much, though, and we can! If we represent everything in a unit sphere, some fun results are incurred. Physics nerds can enjoy this one.

Tree Yoga More fractals everybody, pack it up. Tree fractals, snake fractals, funny yoga pose fractals, we've got it all.

Regular Homotopies in the Plane, Part II! Whitney-Graustein Theorem, but only the back half. We have a method for turning curves into other curves, but sometimes it doesn't work. We prevail and push onwards, though, and mathematics is ours.

Donuts! ... The donuts hate you.

3-Ring Circus The orbit of three rings in an interesting configuration can do interesting, chaotic things. They operate on differential equations, whatever those are. They sound silly, though.