MathILy

Record of Mathematics

Issue 1

serious mathematics infused with levity

Calendar for Week 2 of Root Class at MathILy:

A reminder of our weekday-ly schedule:

Breakfast 8:15–8:55 at Erdman
Lunch 1:05–1:30 at Erdman
Dinner 5:35–6:30 at Erdman
Morning class 9:00–1:00 in Park 336, 337, and 328
Daily Gather 4:30–5:30 in Park 338
Evening Class 7:00–10:00 in Park 336, 337, and 328

Special things:

- On Monday and Tuesday, indicate your interest in various Week of Chaos potential topics.
- On Thursday, receive instructions on reflection/introspection and on Friday before the start of evening class, send completed introspections to your instructors.
- On Friday, read Week of Chaos course descriptions and on Saturday submit preference forms.

Daily Gathers:

Monday: Hannah—Slugs and not-Slugs
  They're sitting and not-sitting on the Fibonacci numbers.

Tuesday: Brian—Fibonacci.
  Period.

Wednesday: Math Movies
  Description.

Thursday: Robert Vallin (Lamar University)—Journey into Induction
  This talk explores my last 30 years, from raw student to experienced professor, and my search for problems that are both fun and interesting.

Friday: Corrine—Let's Take a Walk.
  Walking can be tricky, especially if you are uncoordinated or if there are black holes into which you might fall! We'll try to walk around on some finite things, and perhaps some infinite things as well.

Saturday schedule:

Breakfast 8:15–8:55 at Erdman
Morning class 9:00–1:00 in Park 336, 337, and 328,
  including polyhedral Zome constructions and individual mini-reviews
Lunch 1:05–1:30 at Erdman
Life Seminar 2:00–3:30 in Park 338. Or 349.
Dinner at the dining hall

Sunday schedule:

Look forward to...Week of Chaos!!!

Contents

Near the end of Saturday's class, we played a game where several dots are drawn on the board and two players, each with a different color, take turns drawing lines between any two previously unconnected dots until all pairs of dots are connected. The player who creates a triangle with his/her own color loses. We proved it is impossible to end in a tie if the game starts with 6 or more dots.

Hannah, Corrine, and Josh's Root Class

Andrew and Zhamilya

Linear Algebra

How do vectors work? What points can you reach by summing multiples of some vectors? What is a span of vectors and how can you find it? What possible intersections of two $k-1$-dimensional objects can you get in d-dimensional space? How does this all relate to matrices and systems of equations? We explored these questions and more! Wow! Linear Algebra! *Josh hands*

Moon Tardigrades

Tardigrades, our favorite critters that can survive in space, have decided to slice apart the moon into several regions. How many regions can they get with a given number of cuts, and what are some conditions on the placement of these cuts that give us the maximum number of regions? We generalized the problem to d dimensions, wrote a recursive formula for the maximum number of regions, linked it to sums in Pascal's triangle, and, for $\mathbb{R}^3$, proved that these cuts should be in general position.

Space Elevator

Corrine's play begins like this: the tardigrade space elevator enterprise has an infinite chain of elevator cars with one passenger per car. When a new one arrives, each previous passenger moves from their car to the next, freeing up a new car. With manipulations of this type, and bijections, we showed that $|\mathbb{N}| = |\mathbb{N}^2| = |\mathbb{Z}| = |\mathbb{Q}|$, so the rational numbers are countable. And there are "bigger" infinities to follow.

Sculptures of Snaps and Zippers

In a tree, there is exactly one path between any two snaps. Does every tree with n snaps has exactly $n-1$ zippers? When a sculpture contains a cycle, which sculptures have snappy trees and what algorithm should we use to construct a snappy tree of these particular sculptures?

Systems of Equations

We were introduced to Sage and learned how to use it to find solutions to the systems of equations. We also explored the row operations on a matrix to solve those equations, which include replacing an equation by a nonzero multiple of itself and adding a nonzero multiple of one equation to another equation. We answered such questions as: is it possible to convert every system of linear equations into a system where all equations are combined into one?

Or can you convert it to a system where every equation that's not all zeros has a variable that no other equation has?

Onion oasis

An onion oasis is a set $\mathcal{O}$ of onions together with a set R of peels with certain rules that we created for onion addition and peel addition and multiplication. Based on those rules, we found properties, examples, and non-examples.

sarah-marie, Jason, and Sara's root class

Henry and Alvin

Note-Taking Diagrams

We had to assign two people to take notes on each day. Each person's name and two copies of each day of the week was written on the board in two rows. A collection of squiggly and straight lines were drawn between days and names. By analyzing this, we found a one-to-one correspondence between days and names and proved that they can be found from similar diagrams more generally.

Induction

We were taught how to prove things rigorously with induction, using a base case, an induction hypothesis, and an induction step.

Slicing Space

We examined the consequences of dividing d-dimensional doughnuts with $(d-1)$-flats. This led to a partial definition of general position, a property of a set of $(d-1)$-flats in $\mathbb{R}^d$: that every set of exactly k of them intersects in exactly a $(d-k)$-flat. We extended the definition of general position to include $(d-k)$-flats by describing them as the intersection of k $(d-1)$-flats in general position.

Lucky Ducks on Lava Rocks

Jason's hometown is full of super-intelligent ducks who live in a settlement of nests perched above a lake of lava. They have constructed some roads between their nests such that a path exists between every pair of nests in the settlement. We are planning to build highways on existing roads that won't be destroyed the next time the lava rises. We proved in class that the smallest number of highways required to connect all n nests in any duck settlement is $n-1$. We found and proved the validity of an algorithm to generate such a connection, which we called a *meh* (Minimal Effort for Humans). We then considered a situation in which each road had some number of sharks on it, and we found and proved correct an algorithm to generate a *meh* that requires fighting off the fewest sharks. Later, giant duck-ants (dants), who spread in every direction every day appeared in the settlement, and we looked at the number of them who end up in a given nest on a given day.