RoMV

Branch Week 2

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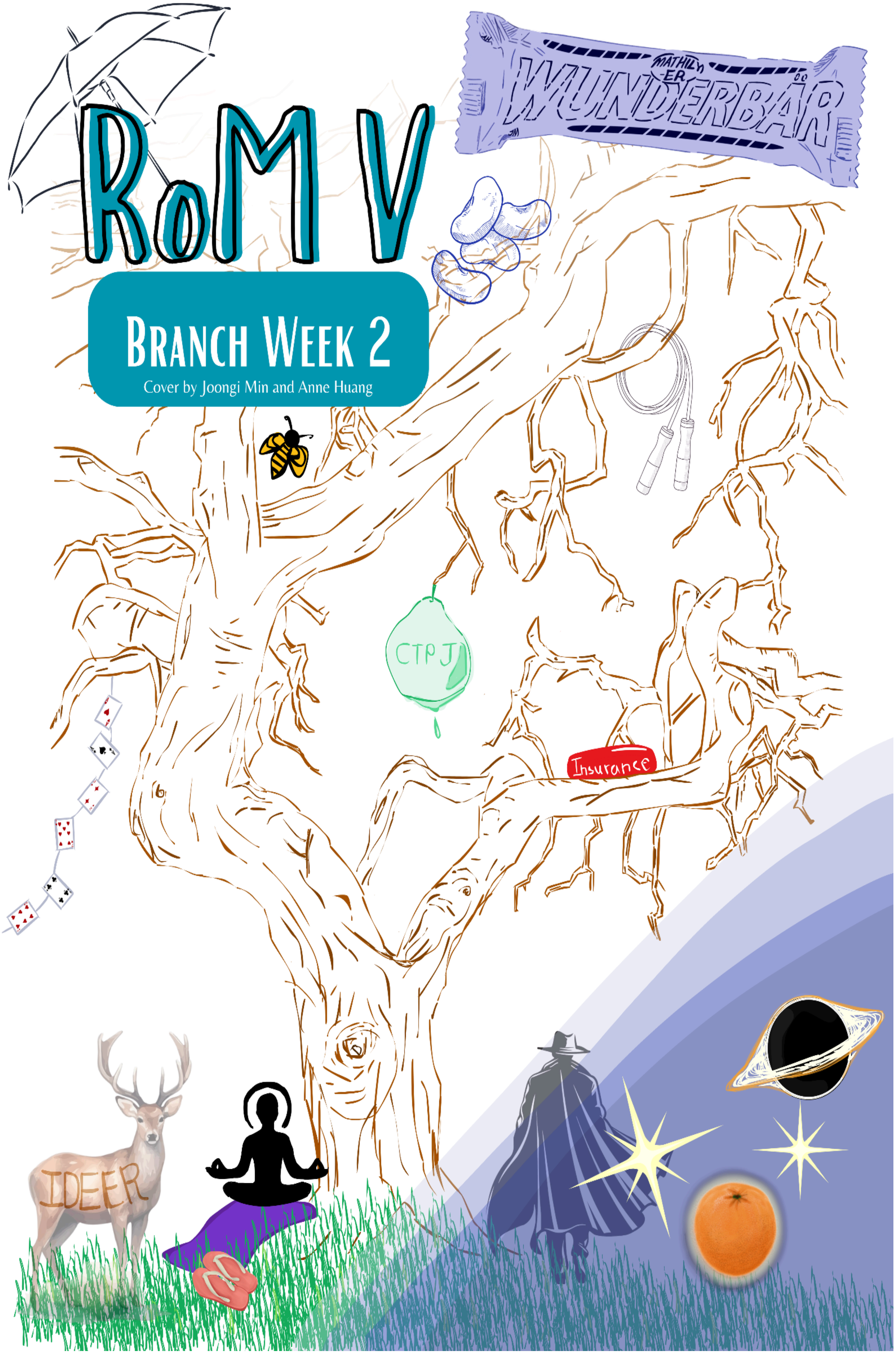

the MathILy-Er Record of Mathematics (RoM)

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In this issue...

1. Class Summaries

1.1 What to Expect When You're Expecting the Unexpected – Corrine, Lixin, Matthew by Lucas, Anna, Ranbeer

Flipping for Freedom

To determine the note-taking assignments for the week, we were asked to choose our favorite sequences (or those left over, as all sequences had to be distinct) of four coin flip results. Then, a fair coin was flipped repeatedly so that when a given sequence of four results appeared, the student who chose that sequence could choose their note-taking day. Once the process had finished, we noted that certain sequences appeared before or more frequently than others.

We first observed that, when finding sequences of length n in a sequence of coin flips, we only need to consider the previous $n - 1$ flips that have happened. We then considered the sequences present at specific moments, called *meditations*. The probabilities of reaching different meditations were visualized through a branch diagram beginning from a "void" state – an empty sequence – and then branching into the prefixes of the desired sequences. We decided to construct *meditation guides* that transform meditations into other meditations with a designated probability. We used these to construct *yoga mats*, which consist of these probabilities.

The yoga mats contained a lot of interesting information! We figured out that we can split it into blocks by the type of meditation. Some meditations cycled forever, which we called *enlightened meditations*, while the ones that didn't were called *mortal* (as a metaphor for the cycle of Enlightenment). In particular, the top-right block (called the *Path to Enlightenment*) told us about the probability of going between meditations. We proved that starting at a mortal state, we eventually get to an enlightened state. Using this, we used Sage to show that for every sequences of length 3, there exists a sequence that appears before it with probability greater than $1/2$, and there does not exist one "best" sequence. We then conjectured that this would hold for sequences of length n.

MathISni Madness

Freshman dorms at MathISni are arranged in an infinite line, and every night, students have to escape their dorms by running infinitely left or right. However, the Director also destroys halls with probability $1 - p$. Can any student escape?

If $p < 1$, no! Infinitely many hallways are destroyed in both directions. What next? Expand! The sophomore dorms are arranged in a grid with infinitely many dorms, and again we ask: can we ensure that a student can escape with some high probability?

We started by defining structures that blocked a student's escape, which we called rings of destruction (RoDs). RoDs are made by drawing the midpoints of destroyed hallways, and connecting bathrooms with a line if either the hallways are parallel or share a dorm room, forming a ring-like shape around the student. We showed that a student cannot escape if and only if they have an ROD around them, and by a counting argument, proved that for $p < 1/4$, no student can escape, while if $p > 3/4$, most students can.

2. Daily Gathers

2.1 Monday – MathIGy Friend Assignments by Amelia

At MathIGy, the staff assigns the number of friends each student can have, or their popularity. Friendships must be mutual; students are always brutally honest about whether they are friends with another student, and no student is their own friend. Students are told of their popularity at Opening Meeting, and they have 24 hours to make exactly that many friends, or none of them are fed for the rest of the program. We explored popularity assignments, or lists of popularity scores of all the students, and friendship networks, which are made to try to satiate assignments.

We found some conditions for a popularity assignment to be satiable by a friendship network: the sum of the students' popularities being even; no one being friends with someone with a 0 popularity score; and the popularity of any given student being at least one less than the number of students with non-0 popularity. These conditions do not guarantee that our assignment is satiable, but if they are not met, our assignment cannot be satiable. Finally, we were able to prove that a popularity assignment is satiable if and only if that same popularity assignment with the student with the highest popularity taken away is also satiable. It's safe to say that friendships at MathIGy are as unpleasant as everything else there.

2.2 Tuesday – Vote for Me: I Can Add Stars! by Aaron S

It's the year 2452, and Noa is running for president. Of course, candidates are ranked based on four characteristics: Charisma, Onesty, Generosity, and Luster. Noa has a genius plan to win the race, which is to add constellations, because the people of the galaxy love adding and constellations.

Constellations are made of stars and frexprs connecting the stars. We want to be able to add two constellations A and B on the same set of stars to get C, a new constellation with on same set of stars as A and B. We denote addition of constellations with a ✯ symbol. We want ✯ to satisfy:

- There exists a constellation I so that $A✯I = A$ for any constellation A
- There exists a constellation B for a given A and C so that $A✯B = C$
- $(A✯B)✯C = A✯(B✯C)$ for constellations A, B, and C.

These are the same as the identity, inverse, and associativity properties of biscuits, making constellations on a given set of stars a biscuit with operation ✯. On a constellation C, we defined C_{ij} to be 1 if there is a frexpr connecting stars i and j and 0 otherwise. Most people defined $C = A✯B$ so that $C_{ij} = A_{ij} + B_{ij}$ (mod 2) for any stars i and j.

People in the future hate odd numbers. Noa defined star-power as the number of frexprs on a given star. Then, a constellation is even if all of its stars have even star-power. We showed that the sum of any two even constellations is an even constellation and (by induction) that any even constellation can be written as the sum of triangular constellations (the smallest even constellation).