the MathILy-Er

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Public Section

Record Of Mathematics (ROM)

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In the Public Section

Monday's Daily Gather

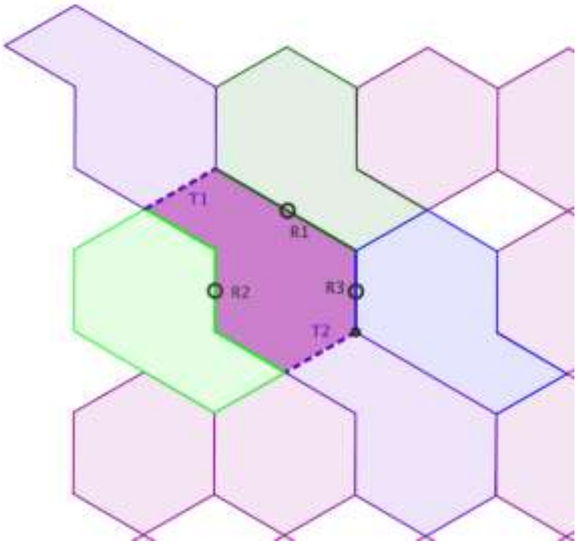
by Aryan

On our first day of the week, we were greeted by our awesome instructor, Alice, for part three of the wallpaper patterns Daily Gather. During Friday's Daily Gather last week, we took and submitted photos of wallpaper patterns around campus that we noticed and thought were interesting. Consequently, Alice's Dropbox had hundreds of photos.

This Dropbox consisted of many tessellations, interlocking patterns, and frieze patterns. Using our categorization skills that we learned over the past two parts of this daily gather, we started categorizing each pattern that we saw into either a frieze pattern or wallpaper pattern. A frieze pattern is a pattern that repeats in only one dimension:

On the other hand, a wallpaper pattern repeats in two dimensions:

Later, we further categorized them using their fundamental regions. A fundamental region is a region in the pattern that can be repeated and reflected to for the rest of the pattern and cannot be broken down into any smaller fundamental regions. Finding the fundamental region for a tessellation was relatively easy. First, we found the lines of symmetry. If the lines of symmetry don't form a closed shape, then repeat a lines of symmetry to close the shape so that it is impossible to further divide the shape with another line of symmetry. Et voila, you should most likely have a region that can be repeated to create the rest of the pattern. Below is an example of a fundamental region:



After finding the fundamental region, we marked the rotational points. Based on this information, we were able to characterize almost every wallpaper. For example, a tessellation whose lines of symmetry could be used to create a triangle whose vertices were rotation points around which six, three, and two rotations, respectively, were possible, would have category *623.

By the end of the Daily Gather, we had gone through all of the pictures, and, thanks to Alice, were able to characterize almost all of them.

Tuesday's Daily Gather

by Alice

Tuesday was Nate's last full day :(To start off daily gather, Nate asked us to figure out what the last digit of 2^{100} was. We determined that since the last digit follows a pattern of 2, 4, 8, 6 and repeats, 6 must be the last digit. After that warm-up, he asked us if we knew what the first digit of 2^{100} was. Jonathan confidently guessed that the number was 1, and was later proven right. We determined different patterns that the first digit could follow after being doubled- for 1, there were five: 1-2-4-8, 1-2-4-9, 1-2-5, 1-3-6, and 1-3-7. Afterwards we counted the percentages of first digits, 1-9, of the first hundred powers of 2. We found out that about 30% of powers of 2 from 0-100 started with a 1. We then used a program to determine $2^{0-10000}$ and obtained similar results. Nate then gave us the inequality $a10^k < 2^n < (a+1)10^k$. We took the $\log_{10}$ of both sides and got $\log_{10} a + k < n\log_{10} 2 < \log_{10}(a+1) + k$.

Nate then introduced us to fractional parts, which is a number minus its floor (the numbers after the decimal point). We then tried to answer his question: does there exist an *n* such that $\{n\log_{10} 2\}$ is in between $\{\log_{10} a\}$ and $\{\log_{10}(a+1)\}$? (A fractional part will be represented by {}) In the end, the answer was yes. Nate explained a theorem from Joseph Pigeonhole that showed it is possible to choose a positive integer *k* such that $\frac{1}{k}$ is less than the length of a chosen interval (I) /2, represented by $\frac{I}{2}$. Ultimately, using fractional parts it is possible to hit every interval of size at least $\frac{1}{k}$. Therefore, since log2 is irrational, our question could be solved. In the end, we learned from Benford's Law (a law derived from the theorem Nate proved) that the percentage of first digits from powers of 2 are reflected by the differences between $log2$, $log3$, $log4$, $log5$, $log6$, $log7$, $log8$, $\log 9$, and $log10$ (which is just 1). These differences are really the intervals we proved in disguise.

Wednesday's Daily Gather

by Clarissa

In an attempt to escape the heat, we held Wednesday's Daily Gather in the dorm's air-conditioned basement. During the Daily Gather, we learned how to create an open-source HTML5 puzzle game on Puzzlescript. Once we entered the program, we saw a blank canvas of possibilities for our puzzle game. Jonah started off by teaching us how to indicate the name, author and homepage of our game. After that, we created objects in the game, including Background, Player, Fire, Goal, and Wall. In the legend, we designated a symbol for each object. This way, we could create each level by typing an array of characters. Puzzlescript also featured a collection of sounds to go along with our actions. We each created our own personalized levels, all with the win condition of getting the "Goal". Finally, we made rules for our game to follow. To start things off, if a player stepped on "Fire", it would turn to "Ash". This simple direction was programmed with LATE [Player Fire] -> [Ash]. This means that once the player is on fire, [Player Fire], the player turns to Ash, -> [Ash].

Our games became more complex, adding fire that spreads with every move with [STATIONARY Fire | no Wall no Fire] -> [Fire | ACTION Fire]. We made our games sleeker by using five by five multicolored graphics to represent different objects in our games rather than simple single-colored blocks. We next added new levels and new challenges in them. Some levels included water-shooting cannons or moving crates. This Daily Gather was informative and a lot of fun. Watch out, Nintendo -- our games are coming for you.

Thursday's Daily Gather

by Robert

On Thursday, we came to the Daily Gather to find that we'd be shown another two person trick. But the best part was that without Alice, Jonah would be doing the whole thing by himself! First, Jonah (Alice) left the room. Next, Jonah (Jonah) flipped eight coins; however, since Jonah (Alice) thought Jonah (Jonah) was bringing the coins, while Jonah (Jonah) was sure he (she) was bringing them, we filled in as the eight coins for Jonah (Jonah). Next, the audience got to flip a coin over, and Jonah (Jonah) flipped another coin over. Finally, Jonah (Jonah) called Jonah (Alice) back into the room to correctly guess which one we flipped over, which was astounding because Jonah (Alice) was gone the whole time.

I could already sense binary trickery pervading the entire process just as normal trickery seeps through Yan's words during a game of Mafia. Because this isn't a magic show, the trick was simplified to two coins to give us the chance to understand its basics. The only other difference was that Jonah (Jonah) flips first. Jonathan and I figured it out very quickly, and decided to show it to the class because apparently that helps the class figure it out more than us telling the class how it worked. I (Alice) left the room, while Jonathan (Jonah) decided which one to flip, and most of the audience (the audience sans Jonathan and me) flipped too. Next, I (Alice) was called in, and told them which one I thought they flipped.

I was only one off.

I blame this failure not on Jonathan (Jonah) or myself (Alice), but on society. See, heads is just considered better than tails. There seems to be no reason, but people like the side of the coin with the inaccurate representation of some old dead guy's face more than the side with the magnificent, patriotic architectural masterpiece. My best piece of evidence is the gameboard for the Game of (Real) Life. Heads for escaping justice, tails for getting arrested. Heads for political change, tails for pepper spray. And finally, heads for male and tails for female. That's just sexist! Real Life has to be the most sexist game we have. Being old is cool though.

Wait, I haven't finished explaining myself.

I (Alice) thought Jonathan (Jonah) said he would make the first coin tails, but he must've said "heads" or "second coin" or something while I (probably not Alice) was contemplating the injustice of it all. When I (Alice) came in, I (Alice) saw the tails in front, and thought the rest of the audience (the audience) had flipped the second coin.

What? No, it's not my fault. I just made a tiny mistake.

Oh, shut up.

After others demonstrated the trick correctly, making the *second* coin tails (so sexist - wait, that was something else), the trick was explained so that we could move on to eight coins. By this time, I sensed Stephen's position trickery as thick as the cement Yan's "Family" uses to weight "swimmer's" legs down, so I suggested that that was the trick. I didn't elaborate, because I had no idea how it might work. But I was right! Aryan soon called out that if you BS added up the positions of the heads (or the tails) after Jonah (Jonah) flipped a coin, you always got zero (unless you counted the last coin).

However, we weren't finished. Jonah said that it connected back to our first Daily Gather, where we couldn't figure out how to halve our chance of death. The problem was that we were all put in a circle, and everybody had a red or blue hat; we can see each other's hats, but not our own. Somebody has to guess their own hat color, and if they're right we all live. But if anybody is wrong, or if nobody speaks, we all die. We realized that we could put eight people in a circle; each would BS add up the red hats (after everybody was assigned a number). If we got our number we'd guess red, and if we got zero we'd guess blue. If we got something else, we'd shut up. Next, we needed volunteers, but only got seven out of eight because half this camp is made of cowards as filthy as the resting places of Yan's enemies would be if they were slathered in tar with the consistency of Yan's worst enemies. However, Jonah joined and we lived!

The End.

Friday's Daily Gather

by Yan

On Friday, Jacob Katz, who is Guidance, Navigation and Control engineer at Space Exploration Technologies, came to us to talk about – surprise – guidance, navigation and control of space vehicles.

First, he told us about guidance. Basically, guidance consists of trying to figure out how to get from point A to point B. Of course, the mechanisms behind it are much more complicated, but this is the essence of it. Guidance must allow for various constraints, such as maximum speed/acceleration or obstacles in the way.

Navigation determines our position and a lot of other factors, such as rotation. There are different kinds of navigation: navigation by compass, navigation by stars, GPS, video cameras, gyroscopes. Without navigation it would be almost impossible to get to the destination, since something unpredictable can always steer a vehicle off course.

To get back on course, we need control. Control corrects navigation errors and counterbalances unaccounted-for factors in order to reach the destination safely.

Then Jacob introduced us to the basics of how control is executed. He uttered unspeakable words such as plant and feedback loop.

Plant is something that is responsible for controlling the vehicle.

Feedback loop is when you take the output and put it back into the plant. It is widely used in control system, since you can calculate where the vehicle will be in a second, and then use that information to calculate where it will be in two seconds and so on.

Let's get down to business:

$x_{t+1}=x_t+v_t\Delta t$

Δt equals 1 second, so we can write the previous equation as

$x_{t+1}=x_t+v_t$, where x is distance and v is velocity.

$v_{t+1}=v_t+u_t* \Delta t$, where u is acceleration and Δt equals 1 second, so

$v_{t+1}=v_t+u_t$

Distance error = (target-x_t) =e_x

Velocity error = -e_x

$u_k=k_pe_k$,where k_p depends on how far are you from your target.

So $u_k=k_pe_x-k_dv$

$v_k=x_{k+1}-x_k$

$v_{k+1}=v_k+u_k$

$v_{k+1}=x_{k+1}-x_k+u_k$

$x_{k+2}-x_{k+1}=x_{k+1}-x_k+u_k$

$x_{k+2}-2x_{k+1}+x_k=u_k$

$x_{k+2}+(k_d-2)x_{k+1}+(1+k_p-k_d)x_k=T$

Let's express distance as a polynomial function, so that $x_{k+1}=zx_k$

$z^2x_k+(k_d-2)zx_k+(1+k_p-k_d)x_k=T$

After dividing both sides of the equation by x_k and raising them to the power of -1, we get:

$$\frac{x_k}{T}=\frac{1}{z^2+(k_d-2)z+(1+k_p-k_d)}$$

This is an equation for a basic control system.

Summary of the Week

by Stephen

Our unit on political mathematics began on Monday morning. The first part of class was focused on a seemingly simple question: What is the best way to conduct a vote? We voted on flavors for pizza, and then began to think about and discard options such as randomness, Yan chooses, and others. We also began to formalize a set of principles to determine whether a given voting system was "fair," such as the Equality Principle (Switching two votes may not affect the outcome of the election), the Deterministic Principle (The same set of votes may not give different outcomes), and others. While a perfect method was not found, several reasonably good ones were. The second part of class was devoted to an equally complex question: how may a pizza be divided optimally? Various algorithms were proposed, but the one that seemed to be the best was Alex's algorithm, which involved splitting the pizza into sections, each person ranking each section, and using ratios to attempt to give as many people as much of the parts of the pizza that they liked as was possible.

The second day began by proving that Mia B. and Jonathan's methods were equivalent. Also, Jerry proved that Clarissa's method is equivalent to them both. Then we defined a 1v1 me bro to be a candidate that will win any 1v1 election against any other candidate. This led to the 1v1 me bro principle which states that a 1v1 me bro should always win an election, a weaker version of Robert's principle. Also, we learned how to write votes effectively. To write votes effectively: write each permutation multiplied by the number of times it is repeated. Then ties were discussed, and eventually the class reached a number of conclusions about ties. Also, we defined a voting system as a function that takes as input a set of permutations of a set of candidates, and may output one or more winners. We then moved on to pizza cutting, and defined a series of principles for "fair" methods and ways and whether or not our methods fulfilled them. Lastly, we began the problem of distributing representatives to states, one of which was percentage based, named Alex's method.

On Wednesday, we began with a discussion of pizza cutting, and, after being given a device that returned a person's value for any part of the pizza, were able to work out a "fair share" method for dividing a pizza among 3 people. Then, we had a very strenuous debate about whether points could have value and whether, if they were on the line of a cut, were duplicated, destroyed, or neither. We then switched over to representative choosing, and analyzed Tom's, Alex's, and John's methods, finding differences in their advantages and disadvantages for specific states, as well as determining whether 2 of them were pwny (had ratios within 1 of the optimal ratios). Lastly, we worked more on voting theory, determining what types of elections a 1v1 me no could and could not win.

Thursday's class began with a discussion of logic and the petropapyrocaesoric vs. 1v1 principle, after which we briefly covered voter preferences in one and two dimensions. Then, we discussed methods of cutting a pizza into n equal or greater sections. Lastly, we continued work on Alex's and Jonathan's algorithms for representative division.

On Friday, we started by proving that Dan and John had the same method for candidate division. We then defined these into a method set, and added various new ways to round in the same set. We then began on SUVYaNs, or Systems of Unequal Voting with Yea And Nay votes. We discussed several governmental examples, and proved that some could or could not be turned into SUVYaNs. Some of these included the U.N. Security Council, U.S. Congress, Canadian Legislature, and European Economic Council. We then tried to analyze whether unequal numbers were equal, and whether given voters actually had any effective power at all.

On Saturday, we began to analyze equations for sufficient votes and determine algorithms for defining relative powers for each voter. We then switched over to bracket-based voting, analyzing several principles and creating a new one, and analyzed cycles of voters and what they do to the possibilities everyone has of winning. Lastly, we discussed several other methods of splitting a pizza among two or three people with vertical cuts such that both people value their part of the pizza equally.