## the MathILy <br> Record of Mathematics (RoM)

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## Welcome...

Our second week at MathILy has been something. We finished Root, and are now totally ready for Week of Chaos with all the math we totally learned. Get excited for this issue, where we include Root Summaries, apology letters, special poems, bingo cards, and complete-your-own-proofs.

## Sincerely,

The Editors
Shlok, Avni, Clàudia, \& Mathes

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### 1.1 Daniel, David, \& Basia by Athithan, Héctor, \& Max

Monxi Negotiations We had some issues with gathering alien secrets.
There are some aliens called Monxi who love to eat fish but will only share their secrets if every Monxi gets a fish they like. We discussed many algorithms, none of which were successful but hypothesized that for every subset of $k$ Monxi, they must like $k$ distinct fish. Later delving back into the origins of CARP we eventually proved conjectures involving satisfying Monxi.

Ponds We further discussed ponds, the main properties of which are having koi, an operation, a ghost koi, doppelgangers for each koi in the pond, and associativity. We found many cool ponds that aren't commutative, including the symmetries of a regular $n$-gon, and the pond of permutations.

Counting Aliens We discussed a combinatorial problem about aliens with all sorts of different properties and solved it using a double-counting argument involving binomial expansions.

Breaking Glass We started by figuring out how to simplify aquariums into leaky aquariums, a form in which we can easily read solutions. In leaky aquariums, the first nonzero entry in each row is 1 , and every other term in that column is 0 . We also showed that any $n$ fish-free vectors build an $n$ screen.
We constructed a way to represent all the points in a MIS using fish-free vectors and variables, which we called a COD, and proved each point's representation was unique.
We continued our discussion on leaky aquariums and proved relationships between MIS-es and screens. We also showed that a COD of a $d$-MIS has exactly $d$ vectors. We also realized that we can choose the constants in the leaky form of our aquarium to be whatever we want by going backward from our desired leaky aquarium since all our video game moves are reversible with more video games.

Fully Smooshed F We played the hit new game Smoosh the F, in which we manipulated a series of points that formed an F using a 2-by-2 aquarium in Sage/Mathematica. Angie then conjectured that if the rows or columns of the aquarium are fishy, the F disappears since it would put the points on a 1-MIS.

Aquariums: the Functionization We proved various facts about aquarium functions, such as composition, the existence of the inverse, associativity, and the conditions on bijectivity. Noam formalized the method for getting a set of solution vectors from a leaky aquarium. We also defined $\operatorname{trench}(A)$ to be the set of solutions for an aquarium $A$. In particular, $\operatorname{trench}(A)=\{\vec{x} \mid A \vec{x}=\overrightarrow{0}\}$.

Sports for Aliens We started contemplating a game between aliens Josiah and Jacob. In this game, each alien took turns drawing a line segment of their color between any two points out of a set amount. Each alien loses if they build a triangle made up of segments only of their color. We proved that the aliens can't tie if more than five starting points are given.

Chain Mail Returns We then considered a cursed email, whose recipients must send to all of their friends every 10 seconds. We then found an algorithm to calculate the number of emails someone in the group of friends has received after any number of minutes.

Niis at (k)Night We also proved that the SNAIL made up of guard towers remaining after pillaging is a WHALE. This helped us count different aspects of Park Sciences and how they relate to each other.

The Aquarium is Everywhere We showed that any environmental function of form $f: \mathbb{R}^{m} \rightarrow \mathbb{R}^{n}$ can be expressed as an aquarium.

David's Socks We learned various concepts in probability such as states, state spaces, events, expectations, and event spaces. We then used these concepts to calculate the expected number of pairs of socks that David brought to MathILy.

The Death and Rebirth of Lillian and Mathes We finished the problem that started it all, which was to prove that for any $d$ and $k$, there exists a set of $k$ finke in a $d$-screen that satisfies the Lillian-Mathes condition. It was a very unifying problem that used pretty much everything we learned throughout root class. It was an amazing presentation that covered every board in the classroom. We also showed that environmental bijections preserve CODs. Then we showed how to find a COD for the school of any vectors using Street Fighting.
Later, we began class by proving the second Mathes-Lillian Conjecture, which related trench and lyceum dimensions to the amount of kelpy and knelpy columns in an aquarium and implied that the dimension of the lyceum plus the dimension of the trench of an aquarium A equals the number of columns in A . This proof was then further used to prove that the dimension of the school of the rows of an aquarium is equivalent to dimension of its lyceum. Following this, we began a discussion on the form of lyceums and trenches in functions and realized that the image of an aquarium $A$ is its lyceum. Furthermore, we showed that A's trench is orthogonal to the school of the rows of A using the Tilapia product. Then, we defined a split screen as a screen translated using a vector not in its school and constructed a way to represent any split screen for dimension $d$ from a $d$-screen and a point in the split screen. Finally, we analyzed TARNs and found out how to calculate the inverse of an aquarium. For the final act, we looked at the fixed points and fixed lines of a 2 by- 2 aquarium function.

Loose Ends We began to tie up loose ends as the end of root class approached. We decided that two SNAILs have an aquatic equivalence between them if there is a bijection between the aquariums that represent their connections or if after mapping each galaxy in the first SNAIL to one in the second, the connections between the mapped galaxies are the same as the connections between the original galaxies. Then, we proved that the rational numbers are surmountable. We also defined two vectors, $\vec{v}$ and $\vec{w}$ to be orthogonal if $\vec{v} \cdot \vec{w}=\overrightarrow{0}$.

