Contents

1 Class Summaries

1.1 Brian's class

Eric

I Hop to IHOP

A l**I**near **H**abitat **O**f **P**athogens, abbreviated IHOP, is a set of vectors, a set of scalars, and two binary operations that we called "addition" and "multiplication" that follows a "short" list of axioms: closure under addition, closure under scalar multiplication, existence of an additive identity, existence of a multiplicative identity, invertibility of addition, commutativity, associativity, distributivity, and existence of a zero scalar. Some examples of IHOPs we came up with were the integers, the reals, n-space, polynomials, and functions. We were able to come up with a more efficient way of checking if a proposed IHOP was valid by defining a sub-HOP, an IHOP whose set of vectors was a subset of another IHOP with the same set of scalars. We showed the sub-HOP needed to meet three criteria: 1) it is not empty; 2) it is closed under addition; and 3) it is closed under multiplication. We also showed the IHOP $\mathbb{R}$ with scalars $\mathbb{R}$ had no proper sub-HOPs.

Hell's kitche(n)

A kitchen is a set of chefs and dishes where each chef is qualified to cook some subset of the dishes. A working kitchen is a kitchen where every dish can be and is cooked by a qualified chef and every chef cooks a dish, otherwise, what are we paying them for? We quickly deduced that every working kitchen necessarily contains n chefs and n dishes, a kitche(n). We also developed a CWaim: every kitche(n) such that all subsets of i chefs cooks i dishes is working. We called this condition the CWaimdition. After three days of work, we were able to prove that all kitche(n)s satisfying the CWaimdition are in business, with a little help from Minickey-D (the second floor kitche(i)) and a kitche($n+1$).

When's a Line not Linear?

We defined a function f to be linear if for all x, y in the domain of f and α in the set of scalars, $f(x+y) = f(x) + f(y)$ and $f(\alpha x) = \alpha f(x)$. Using this definition, we discovered that a line of the form $f(x) = x + \alpha$, $\alpha \neq 0$, was not, in fact, linear. We were also able to show that transformations such as translations, reflections, and rotations were also linear. Additionally, we showed that matrices represented linear maps and defined matrix multiplication to correspond with the composition of linear maps.

SET Theory

We were tasked with finding the expected number of SETs on a table with twelve randomly dealt SET cards. We first showed that given any two SET cards, only one unique card could complete the SET. Using this lemma, we found that the number of expected SETs was $\frac{220}{79}$, or roughly three. We then defined expected value and formalized probability.

A Blessing of Narwhals

There are Definitely Infinite Narwhals Eating bReakfast, a DINER, under the ocean. Every narwhal is antisocial and seated at a different table labelled 1,2,3, ... Another narwhal arrives to eat breakfast, but all the tables are full. We showed that by shifting each seated narwhal down a table, the first table is now open for the new narwhal. But suddenly, a blessing of infinite narwhals arrives! The manager of the DINER is not having a good day, but luckily we found a way to seat the blessing as well. The narwhals, having been seated, are unsure of what to order. They decide to form committees to vote on which food is best. Each committee needs a table, but unfortunately for the manager, we showed it was impossible to seat all committees in our DINER.

Frankenvine

Our polygonal grapevines where every grape is connected to all others have tragically died and resurrected as horrible zombie vines, neither alive nor dead. Our archnemesis Nairb and the evil garden witch take turns withering and revitalizing stems on our horrible vine. However, if either a triangle of death or a triangle of immortality (which is depressing) is formed, the horrible vine is lost forever. We showed that if the horrible vine had six or more grapes, it was impossible for the vine to live happily ever after.

Breakfast Polls

Brian polled a subset of everybody at MathILy about what they ate for breakfast (blueberries, bagels, donuts, and/or pancakes) but forgot to record who he polled. Luckily, he recorded the numbers for each type of breakfast. From this, we were able to determine the total number of people he polled and were able to generalize by developing a formula for the cardinality of the union of n sets.

I Have a ClaIm

Defining C(f) and Im(f) to be the cured set and image of some linear map f, respectively, where the cured set is the set of all vectors such that $f(v)$ is the zero vector, we ClaIm-ed that the dimension of C(f) was equal to the number of non-Root-of-Happiness columns and the dimension of Im(f) was equal to the number of ROH columns in the row-reduced form of the matrix describing f, respectively. We were able to prove this ClaIm after a few days of work and showed that dim(C(f))+dim(Im(f) was equal to the dimension of the domain of f. We also finally formally defined dimension in a number of ways, such as the minimum number of vectors in a set necessary to define the entire set through sumultipling (adding and multiplying).

Garden Walks

Taking a grape-stem configuration, a garden, we wanted a way to represent the garden in a way digestible by a robot. We came up with the idea of an adjacency matrix, where elements in the matrix are either 1 if the grapes corresponding to the row and column are connected and 0 otherwise. Unfortunately, multiple matrices could represent the same garden. We claimed that if the rows and columns of a matrix could be manipulated through swaps to another, the two matrices represented the same garden. We also found a way to count the number of paths of length k between any two grapes by raising the adjacency matrix to k.

Form of... Ice!

In quite possibly the most confusing abuse of defining terms since the inception of MathILy, we defined two spaces as ice-o-morphic if there existed a well-defined, bijection, an ice-o-morphism, between the two spaces. We showed that two ice-o-morphic spaces had the same dimension. However, we then defined two gardens as ice-o-morphic if they shared the same set of matrix representations and two TIM HORTONS as ice-o-morphic if there exists a bijection that satisfies $f(a \cdot b) = f(a) \cdot f(b)$ between the two TIM HORTONS.

Brian's Toys

Brian had fond memories of a pentagonal toy that could be put away in a pentagonal container cemented to the floor of his closet (for unknown reasons). We showed that there were ten ways to put the toy back using only rotations and flips along an axis. We were also able to construct a TIM HORTONS describing the state of the toy and the rotations and flips performed on it. However, Brian also had a Zometool tetrahedron that he studied the shadow of. Using DINER architecture, we showed that the number of vertices plus the number of faces minus the number of edges equalled 2.

1.2 Hannah's class

Sebastian

The last week of Root class is over, and while I can't wait for the week of chaos, I have to admit the last two weeks were incredible. From duck swamps and looking for sepses in slimes to shark bulldozers and gangs made of an old lady and a keep left sign, a lot has happened in just the few past days. Also, if any of the above sound strange to you, then you are not yet a true MathILy student.

The topics this week expanded on the areas covered before, especially in the area of linear algebra. We got to know swamps and slimes last week, but after this work, it now becomes clear that we were merely scratching the surface of the topic last week. Linear functions were introduced at the very beginning of the week and we continued to use them repeatedly (they are important after all). We learned how to express them as matrices, which in all likelihood, complicated the topic even further. Either way, it was useful. This led us to explore various properties of such matrices. For example, we described Eigenvectors, where for an $n \times n$ matrix A they are such vectors $v \in \mathbb{R}^n$ that there exists some $\lambda \in \mathbb{R}$ such that $Av = \lambda v$. The corresponding λ is then called the Eigenvalue. The discussion about vectors, which started with an army of tiny Roombas™ last week, continued. However, the theme shifted from cleaning robots to a more medical area, as we defined a sepsis: a clean set of landmark vectors for an arbitrary slime such that their film is equal to the slime. As we learned from convenient anagrams: She Impassively Erases (or, if you are a boring person and do not want to decode this, Every Slime Has A Sepsis).

Another area we have continued exploring were graphs. From demolishing the castle of Queen Neptune to sending spam to all your friends, all of those problems seemed similar and vastly different at the same time. The second one is particularly interesting, as our solution to it used matrices-something that seems unrelated, but as it turns out, solved the problem. We tried applying that technique to a problem where we had to distribute food to people with various preferences. We encoded those in a matrix with 1 for liking and 0 for disliking and it seemed to be going well until different numbers for the preferences started to appear. We had no idea what to do so we had to look for a different approach.

The last of the general areas we covered was combinatorics and expected value. This one produced quite a few problems that had many of us scratching our heads for a long time until we eventually figured out the answer; it was always far simpler than what we thought. We solved the "famous cereal coupon problem" where every day we buy a box of cereal containing a coupon. There are a total of n types of coupons, each with the same probability of being found. The question is: what is the expected values of days it will take to get a whole set? This seemingly simple problem turned out to be quite complex, and quite fun. Try it for yourself, it is famous for a reason.

It's really strange to think that there will be no more Root classes. They were a fantastic opportunity to explore different areas of math, all of them interweaving in unexpected ways. I will surely miss those times in Root, when everything fell into place and it became clear the topic were not disjoint (like when the dimensional cutting problem presented itself inside the Pascal's Triangle or Winston's Lemma popped up in the middle of nowhere). Now all we can do is sit back and enjoy the Week of Chaos.

Also, goodbye Connor! Thanks for being such an awesome instructor and being so cool to talk to.

1.3 sarah-marie's class

Edwin

Wolnets

Just as the alien government(the class) had solved the problem of connecting planets and were about to implement the solution, they realized that different woles have different costs! (This is not a great government.) In a brave attempt, the alien government developed two new methods, called Method 2.0 and Method 5.94 (the papers for Methods 2.01 through 5.93 were lost in the hyperquake). Then the alien government made Method 6.2 (Don't ask about Method 6.1. No one knows) and showed that it workeed. The Hilbert Hotel An infinite hotel has many interesting properties, like everyone being able to get as much food as they want, and full capacity not meaning no more people allowed in (in fact, an infinite amount of people can be let in, and an infinite amount of infinite people.(This hotel must be infinitely rich. . .) But then, a man named Russel tries to make clubs of people! In fact, every possible club of people. Each club must meet in a room and only one room, and a very conniving someone creates the absentees club: the people who are not in the club that meets in their room, and the system Russel makes breaks down.

Rargrors and Final Frontiers

Ragrors are sets of things that satisfy a plethora of properties (say that 3 times fast). Upon further inspection, we discovered that this has many connections to other topics in class! (who would'a thought . . .)

Final Frontiers are also sets that satisfy a (different) plethora of properties. We showed that there exists some linear map between any two Final frontiers (wowee!)

Sorcerers and Magical Objects

In a set of n sorcerers, each sorcerer has a subset of n magical objects that he/she likes. Our goal is to match each sorcerer to an object that he/she likes (naturally, sarah-marie gets the Hagoromo chalk, the only object that can channel a mathematician's power). Upon learning of them, we threw conjectures at it like Nadav throws ninja stars (that is to say, about 4 of them, quickly, and not accurately, although Nadav would disagree.) Claim 2.0 and Jake's Way were starting to gain approval, Adrian slayed them both with one counterexample! (Dangit Adrian . . .) But eventually, we still showed when it was possible and when it was not, using a neat severance argument.

Linear Algebra

We considered polynomials and tarmices (and anyone who mispronounced tarmix as matrix was yelled at) and how to define the multiplication of two matrices (TARMICES dammit) and once we did that, we could toy around with the idea of inverses. We also considered functions and defined functions of those functions called the image (reasonable) and popcorn (not so reasonable) and made some interesting conjectures. We also showed that any operation to a tarmix can be written as the product of our original tarmix and another tarmix.

Planarian Bakery

A bakery run by planarians has rooms filled with cake and ingredients to make cake and things to bake cake. Unfortunately, their bakery is on a small island and sharks (who like cake because everyone likes cake) can break through walls of rooms and attack the bakery. We made some conjectures about the bakeries (and the ways sharks can destroy them), and eventually proved them.

Poofing Pills

Brain Force Plus pills go poof in groups of n and can be bought in packs of k. Nadav wants exactly one pill (for science! *cough*). But then there was a sale where you could buy any number of Brain Force Plus pills as long as you buy the same number of DNA Force Plus pills (I guess they give you more DNA?? I have no idea). Nadav again wants exactly one of each(For science! *cough*) We showed when these are possible (and when they aren't).

The Donuts

The donuts spin in unison. Some of them have sprinkles. Some are without sprinkles. We tried to relate the set of donuts with some characteristics to each of the sets of donuts with a certain characteristic. Two methods to find the same relation emerged: one succeeded, and one flopped on the floor and died.

Shadows

We considered shadows of convex solids on a plane, and we found that it was related to planarian bakeries! (Wow, all these topics are related . . .) On the way there we proved some conjectures and disproved others, and also proved many seemingly unrelated things on our way there.

TAKE OVER THE WOLNET!!! A planarian is intends to colonize a galaxy(wolnet). Each turn, it splits itself into parts such that each part goes to a different neighboring planet. We explored how to find the next turn given the last one using tarmices (and had fun colonizing wolnets).

Expected Values

We defined functions for probability, expected value, and used them to show that expected value functions are linear (which leads to some unexpected applications and some surprised expressions).

Countable Infinities (and Uncountable ones too)

We showed that some infinities are bigger than others, using a similar argument to the Hilbert Hotel, and by showing that no bijection existed between the reals and the naturals.

Additionally, Nadav is leaving after this week (oh no ☹). Thanks to Nadav for being an amazing instructor for the entirety of Root Class (and a lethal CTF player outside of class). We will miss you!!

2 Daily Gather

2.1 Monday

Connor Halleck—Dubé's Chomping Integers Jake Shannin