1.1.4 Linearity

How can the term "linear" be defined? sarah-marie asked this unexpectedly challenging question to the students. Following the students' rather unsuccessful discussion (including Joel's bad definition of a plane), sarah-marie defined a hyperplane and the two main properties of *linear* functions. The students also created more precise definitions of planes and lines as well as examples of linear functions.

1.1.5 Giraphs

A giraph is defined as a finite set J of joints and a set N of pairs of joints, called necks. Further, for any joint in J, there must exist a path from that joint to any other joint in J. Giraphs were a major part of the first week of sarah-marie's root class.

Some Plants are also Giraphs Lisa defined four kinds of plants: a winterplant is a giraph in which every neck disconnects; a vine is a giraph containing no cycles; an elderberry bush is a giraph with the maximum number of joints for some number of necks; and a stinging nettle is a giraph with the minimum number of necks for some number of joints. The students tried to prove that $\{\text{winterplants}\} = \{\text{vines}\} = \{\text{elderberry bushes}\} = \{\text{stinging nettles}\}$ but only succeeded in proving $\{\text{winterplants}\} = \{\text{vines}\}$.

When Giraphs Eat When a giraph gets hungry, it wants to eat the most filling vine possible. What is the largest vine (with size defined by number of joints) that a giraph can eat? Students proved that the most filling vine includes all joints of the giraph.

Giraphs in the Hospital One day, a giraph was walking along when the wind began to blow severely. This caused the giraph to become pendulum-y and consequently break all of its necks, so the giraph was rushed off to giraph hospital. Each neck costed a certain amount in giraph currency (GC) to fix. The giraph only had two requirements of the doctor: 1) that it stays a giraph, and 2) that it keeps all of its joints. What is the least amount of GC that the giraph will have to pay to repair itself?

Interesting Proofs about Giraphs Some interesting conjectures were proven about giraphs, including that all vines with at least two joints have leaves and that any vine with n joints has $n-1$ necks. Other problems on the problem sets involved a variety of plant giraphs, such as flowers.

Fortresses In the middle of the ocean on an island on a planet that is not Earth, the Mathelonians built a fortress to hide their precious book from the SIG. This fortress was constructed with walls and corners, with no wall crossing another. Unfortunately, the Mathelonians have forgotten where they hid the book, but they still want to protect the book from the SIG. By using a probe that can go through one wall at a time, they can cloak a room they enter to hide the book. In a similar situation, the SIG are trying to find the book and they can break down walls one at a time. Are fortresses giraphs? Are giraphs fortresses? Students attempted to prove that all fortresses are giraphs, but not all giraphs are fortresses.

1.1.6 The Number Devil/Pascal's Triangle

The tale of Robert and the Number Devil presented some interesting patterns in Pascal's triangle, some of them being: the hockey stick trick, fibonacci sequence, sums of rows being powers of two, and the odd numbers forming the Sierpinski triangle.

1.1.7 Thingspaces

The concept of thingspaces was introduced early in the week. The class contemplated what exact properties a thingspace needed to satisfy, deciding very early on that Nates potato wasteland did not qualify as a thingspace. The students were able to ultimately decide on seven axioms, in addition to closure of addition and scalar absorption, that would define a thingspace and further created two properties that may or may not hold for thingspaces.

Thingspaces vs. Thingdules Thingspaces must have a scalar set for which division works, while thingdules do not necessarily have such a scalar set.

1.1.8 Line Duels

Find a partner to fight with. Draw 5 or 6 points, and choose a different color for each player to draw in. Each player draws a line connecting two points (that are not already connected) such that you do not create a triangle in your own color.

Is there a general strategy to win? One strategy that was presented was to try to force a draw.

Can there be ties? It was conjectured and proved that there must be a winner if you played with ≥ 6 points.

1.1.9 Faucet Coordinate System

sarah-marie's frustration with poorly designed faucets (particularly the ones in the dorm showers) led to a discussion about different types of coordinate systems—for this specific situation, she wished to find a coordinate system to define the volume and degree of the water from the faucet. Students created a coordinate system in which the volume of hot water was on one axis, and the volume of cold water was on the other. This example of a coordinate system led into a more general discussion about coordinate systems.

1.1.10 Linear Combinations and Linear Functions

A linear combination is defined as an expression constructed from a set of terms by multiplying each term by a scalar and adding the results. Students attempted to prove that a function between thingspaces is linear if and only if it preserves all linear combinations.

1.1.11 L<u>in</u>dep and Lindep—Linear Independence and Linear Dependence

If a set of things is linearly independent, this means that no member of the set can be expressed as a linear combination of other elements of the set. Alternatively, if a set of things is linearly dependent, this means that at least one member of the set can be expressed as a linear combination of other elements of the set.

In order to better define both l<u>in</u>dep and lindep, sarah-marie proposed some questions to the students:

- How do we know that $(1, 0)$ and $(0, 1)$ are not equal?
- How do we know that $(1, 0)$ and $(0, 1)$ are linearly independent?
- How do we know if some x_1 and x_2 are linearly independent?

The students began answering these questions but have yet to create better definitions for the two terms.

1.1.12 'Jections

Three basic definitions used to describe functions were defined: injective, surjective, and bijective. A function is one-to-one, or injective, if no two values in the domain will result in the same target value. A function is onto, or surjective, if every value in the target is "hit" by some value in the domain. A function is a one-to-one correspondence, or a bijection, if it is both one-to-one and onto. sarah-marie also explained why "range" is an evil word, and why it is not the same thing as the target, or codomain.

1.1.13 Gaussian Elimination

After long pondering, the class had a revelation about how to solve systems of equations in a systematic way. Gaussian elimination is a step by step procedure using a matrix to solve the system. This process is very useful with linear combinations. However, sarah-marie prefers that we never do Gaussian elimination by hand, while Nate makes us do it for his enjoyment.

1.2 Tom's class—Bill, Chalklate, and How Raspberries can Become Bananas (Cynthia & Laurel)

To kick off the program, we solved a puzzle to assign note takers for each day of class. The puzzle contained the names of the students with a line from each name drawn to a day of the week that we could take notes. However, multiple lines were drawn to connect pre-existing lines in a seemingly random fashion, which we eventually discovered directed an algorithm to assign note takers. For those who weren't familiar already, we went over ideas behind inductive proofs, which helped us prove countless conjectures and claims throughout the week regarding topics such as the Towers of Hanoi and trees (in graph theory). We explored our conceptual understanding of Cartesian planes and coordinate systems by explaining such concepts as points, axes, lines, and vectors to Bill the Turtle.

Bill the Turtle appeared later in the week when we explored linear equations across different vector spaces, as we had to come up with a definition of linear equations that would have a consistent and predictable *turtle value*. We also investigated defining real spaces using vectors in the "turtle smokin' crack" problem, where Bill was a bit high and thought that the three vectors $e_1 = \langle 1, 1 \rangle$, $e_2 = \langle -1, 2 \rangle$, and $e_3 = \langle 2, -2 \rangle$ could be used to define any point in $\mathbb{R}^2$ space. We showed that for some point (x, y) in the plane there were multiple ways to combine the three vectors to reach (x, y). Since each point was not uniquely defined, these vectors did not define $\mathbb{R}^2$.