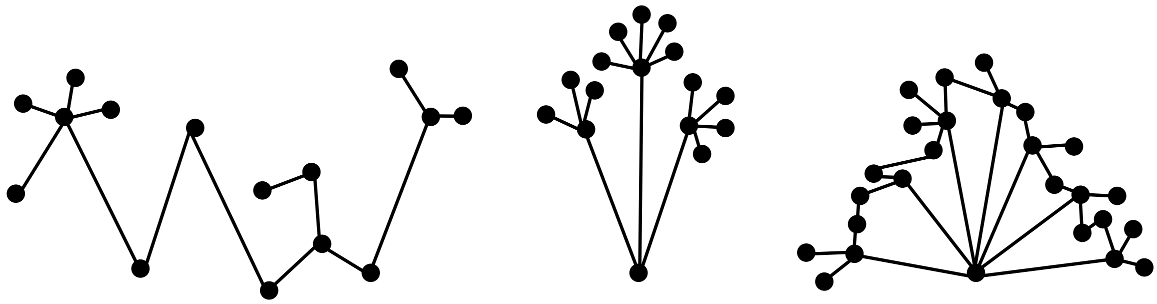
Trees Worksheet



Two trees and a non-tree.

1. Draw five different trees, at least one of which has fewer than 4 vertices and at least one of which has more than 12 vertices.

2. Count the number of vertices and the number of edges of each tree you drew, and of the example trees. Do you see a pattern? If so, what is it? (If not, look harder.)

3. Just for good measure, count the number of vertices and the number of edges for at least one non-tree. Does the pattern hold for non-trees as well?

4. Try to prove that a tree with n vertices has ____ edges. Make sure you understand why this statement is true, but don't get too hung up on a formal proof if you're not making progress—it's a bit technically tricky.

5. Can a connected graph with n vertices have fewer vertices than a tree? Give an example of such a graph or prove that none can exist.

Spanning Trees Worksheet

1. Draw a connected graph with at least 8 vertices that is not a tree.

2. Give an example of a subgraph of your graph that is not spanning.

3. Give an example of a subgraph of your graph that is a spanning subgraph but not a tree.

4. Now find a spanning tree for your graph.

5. Show that every connected graph has at least one spanning tree by giving an algorithm for finding one.

6. Did your algorithm begin with just the vertices, or did it begin with the whole graph? Find a second algorithm that begins differently than your first.

7. Prove that your algorithms work. That is, show that the output is a tree and that the tree includes all the vertices of the original graph.

8. Show that every graph, connected or not, has a spanning forest.