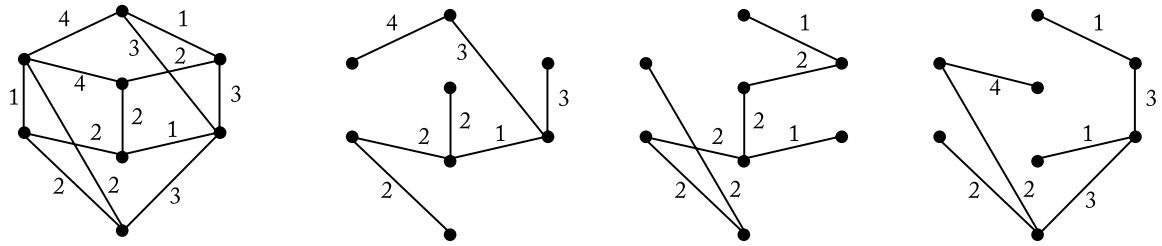
Weighted Spanning Trees: Optimization

Suppose you want to create a high-speed computer cluster from existing machines. In order to do so, you will need to network the computers together using actual physical cables for maximum speed; and, you need to do this networking cheaply. (This idea generalizes to many other situations, such as laying oil pipelines or designing subway systems.) We model the computers as vertices and the possible cable connections as edges, and label (weight) the edges with cable costs.



A weighted graph and three weighted spanning trees.

1. Compute the total weight of each of the spanning trees shown above. Which has the smallest weight? Is that the minimum possible weight? If not, construct a minimum-weight spanning tree.

2. Develop an algorithm for finding a minimum-weight spanning tree in a connected graph.

3. Did your algorithm begin with just the vertices, or did it begin with the whole graph? Find a second algorithm that begins differently than your first.

4. Try to prove that your algorithms work, in the sense that (a) they produce trees and (b) they produce a total weight that is smallest. (This is pretty challenging, so do not be surprised if you can only come up with part of a proof.)