Spanning and weighted spanning trees

A different kind of optimization

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Definitions and examples
Graphs

A graph is a collection of vertices (that look like dots ●) and edges (that look like curves ——), where each edge joins two vertices. (Formally, a graph is a pair $G = (V, E)$, where $V$ is a set of dots and $E$ is a set of pairs of vertices.)
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Here are a few examples of graphs:
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Here are a few examples of graphs:

Two vertices joined by an edge are called adjacent (see $a$ and $b$). Two edges that meet at a vertex are called incident (see $e$ and $f$).
Subgraphs

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Here is the second graph, shown as a subgraph of the fourth graph.
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courtesy of dr. sarah-marie belcastro, http://www.mathily.org
Trees

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A *cycle* is a sequence that alternates between vertices and edges, and whose only repetition is the first/last vertex. A cycle is shown by itself as the top part of the left graph above.

A *tree* is a graph that is connected and has no cycles. One is shown to the right above.

A *forest* is a bunch of trees.
Spanning Trees

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Here are spanning trees of the above-pictured graphs:
Weights are labels on the edges and/or vertices of a graph that often denote costs or distances or energies. Here’s a weighted graph:
Weighted Spanning Trees

The *total weight* of a spanning tree is the sum of the weights on its edges.
A *minimum-weight* spanning tree is one that has the lowest possible total weight.
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Here are a weighted graph, a spanning tree of total weight 6, and a spanning tree of total weight 7; are either of these minimum-weight spanning trees?
Time for Worksheets!

No, really. It’s time to work on worksheets now.
Final notes: MathILy

- intensive summer program for super-smart, super-cool students
- extremely interactive and silly and inventive classes
- discrete and applicable college-level mathematics
- Root class, then Week of Chaos, then Branch classes

http://www.mathily.org