# Spanning and weighted spanning trees

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Definitions and examples

# Graphs

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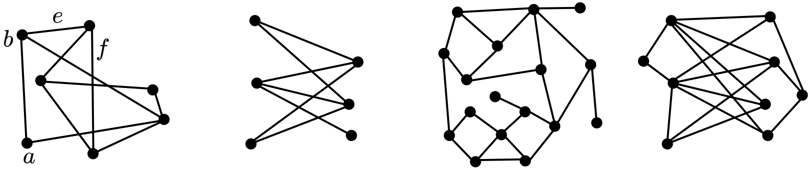

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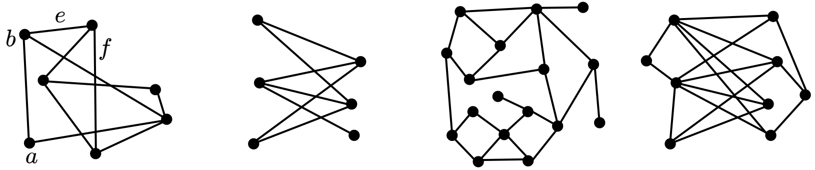
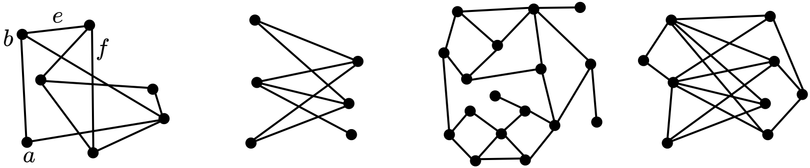



Two vertices joined by an edge are called *adjacent* (see $a$ and $b$).
Two edges that meet at a vertex are called *incident* (see $e$ and $f$).

## Subgraphs

A *subgraph* is a graph that is contained within another graph. For example, here the second graph is a subgraph of the fourth graph.



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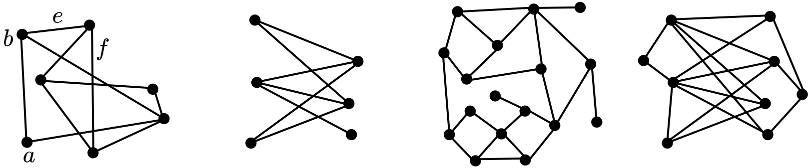
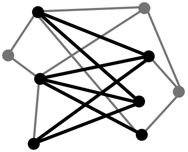
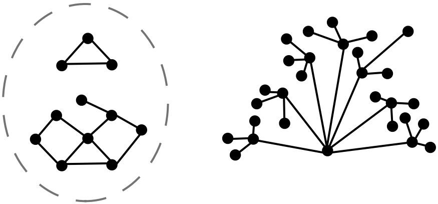



Here is the second graph, shown as a subgraph of the fourth graph.



## Trees



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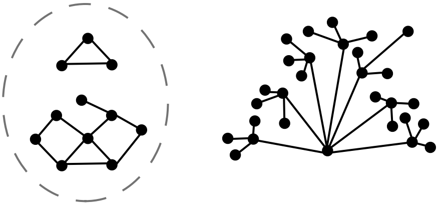

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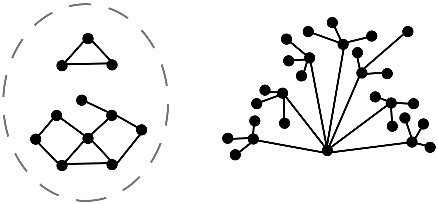

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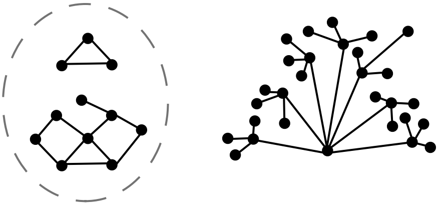

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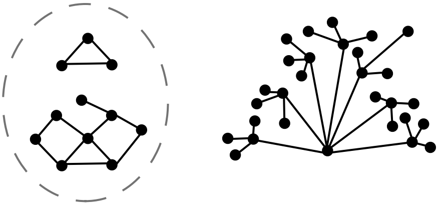

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A *tree* is a graph that is connected and has no cycles. One is shown to the right above.

A *forest* is a bunch of trees.

## Spanning Trees

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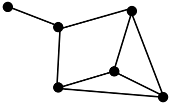

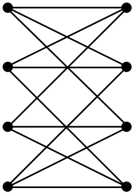

Here are spanning trees of the above-pictured graphs:

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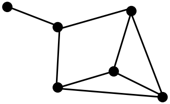

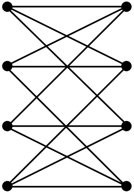

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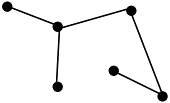
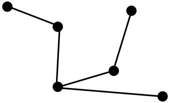
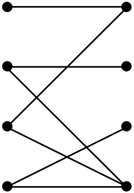
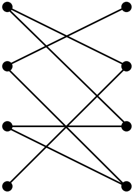


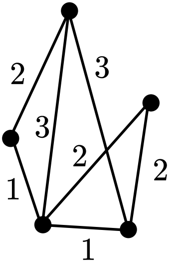






## Weighted Graphs

*Weights* are labels on the edges and/or vertices of a graph that often denote costs or distances or energies. Here's a weighted graph:



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The *total weight* of a spanning tree is the sum of the weights on its edges.

A *minimum-weight* spanning tree is one that has the lowest possible total weight.

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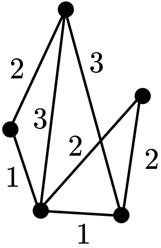
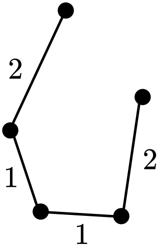
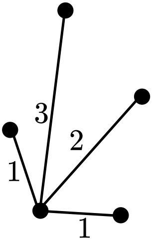
Here are a weighted graph, a spanning tree of total weight 6, and a spanning tree of total weight 7; are either of these minimum-weight spanning trees?



# Time for Worksheets!

No, really. It's time to work on worksheets now.

## Final notes: MathILy

- intensive summer program for super-smart, super-cool students
- extremely interactive and silly and inventive classes
- discrete and applicable college-level mathematics
- Root class, then Week of Chaos, then Branch classes

`http://www.mathily.org`