Week of Chaos
RoM
Week 3_MathILy, 2023
Rob
it's MÖBIN time
Beezzzzzt's Theorem
MAX
MIRIAM
E(ℓ) = ∞
C=E(m_b,K)
||·||_p-FRANK
Gabe
LIMEY
WANTED
DG
LEFTY
RIGHTY
ETHAN
ALLEB
R(3,4) ≤ R(4,2)
BOGUS!
INCORRECT!
WRONG!

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Welcome…

It has certainly been a thrilling *Week of Chaos*! We have just wrapped up all of our 26 *chaotic* classes, and wrapped up a week of crazy games and fun excursions! Prepare yourselves for spectacular recaps of Week of Chaos classes, peculiar quotes, some Canadian-ish fish, and an evocative movie review.

Sincerely,
The Editors
Angie, Cathy, Felix, & Vicky

In this issue…

1.1 Groups and Graphs

BY Adam and Prince, taught by Daniel

Daniel started class by talking about the immense rain on Sunday, which caused most of us to be confused. Turns out, he found a way to connect the immense rain to the class, because apparently, in some yokai/constellation/WHALE world, it is raining at a specific star and the rain continues to spread along the WHIMs. We already ran into a problem: the terminology was incredibly confusing, especially for a class that was named in the most straightforward way possible. At this point, Daniel gave up and listened to Robert's suggestion of using "real" terminology. SPOILER—a *graph* is a yokai/constellation/WHALE and a *group* is an alchemy kit or whatever (you get the idea).

We then used a group and its elements to form a graph. To do this, we represented the elements as vertices and represented the operations as edges. However, as expected, we once again gave up on the new terminology and went back to the old terminology for better understanding. Despite this, we did end up defining some new terms. For example, a *flowchart* was this "graph" or yokai or whatever, and the *generating set* was the set of operations that we used to get the stomachs/WHIMs.

To proceed, we experimented with how generating sets change flowcharts (which it does quite a lot depending on how different the generating sets are). However, once we got this part, another challenge awaited us. We considered a scenario where Gabe and Alan (yes, Alan even though he wasn't part of our class) are on a flowchart and are trying to reach each other after some amount of flooding. Can they do it? Turns out, it depends a lot on which alchemy kit the flowchart represents.

The results: the answer depends and is quite wildly based on which alchemy kit we use, but it somehow doesn't matter for the generating set. Specifically, Gabe and Alan will remain separated, but they will always be able to reach each other in $\mathbb{Z}^2$ over addition. Based on these conclusions, Daniel suggested a different approach: trying to find the disconnected n-tuples of students. This meant that we had to try to find the maximum number of students that could be separated from every other student for a given flowchart (or alchemy kit, to be more specific). This way, the best "disconnected n-tuple" for the first alchemy kit mentioned in this paragraph would be a 2-tuple, and for the second one, it would be a 1-tuple. To make life easier for us, we defined the aquatic severity of a flowchart as the maximum n-tuple that can be disconnected. Finally, using all our knowledge, we were able to show that the *aquatic severity* is either 0, 1, 2, or ∞, which was a crazy discovery that took a lot of time and effort.

1.2 The Geometry of Mordor

BY Teddy, taught by Tom

Dobby and Winky Elven siblings Dobby and Winky recently moved to Gandor. While their human-tongue is pretty good, Winky mixed up the meanings of point and line. Will this cause trouble in their geometry class? Not necessarily. It turns out that we can exchange lines and points in some geometric theorems, such as Desargues's theorem. We then investigated Desargues's theorem in GeoGebra, and finally proved it by changing our perspective from 2D to 3D.

Perspective drawing Parallel lines do not always appear to be parallel in the real world. In perspective drawing, parallel lines will intersect at the vanishing point, which is "infinitely" far away from our painting. In Thursday's class, we drew and proved this result using homogeneous coordinates.

Sauron's canvas With his all-seeing eye, the artist Sauron is at the origin, looking forward to drawing the landscape on the subject line $y = 1$ to the canvas at $y = x - 1$. In order to help Sauron complete his painting, we connect a point on the subject line with Sauron's eye by a line, which intersects the canvas at the location for the point to be drawn. But there are points that Sauron is not able to draw… or are there? What will happen if we consider the points at infinity?

Homogeneous coordinates In projective geometry, we use homogeneous coordinates to represent an equivalent class of points/lines. For instance, the homogeneous coordinates (a, b, c) do not represent a single point. Rather, it represents the parametric line $k(a, b, c)$, obtained by connecting the point (a, b, c) with the origin and extending the line. Similarly, a "line" in homogeneous coordinates actually represents a plane, denoted by $[a, b, c]$ if it has the equation $ax + by + cz = 0$. We found that points at infinity can be described by a line using homogeneous coordinates.

Power of projection Tom introduced Pappus's theorem to us in the last class. We needed to show the three points of intersection in the theorem are collinear. We tackled the problem by projecting two intersection points to infinity, which created two pairs of parallel lines in the graph. This greatly reduced the difficulty of the proof, and it was straightforward to show that the third intersection point also mapped to infinity, thus completing the proof. Finally, Tom demonstrated the concept of duality by showing that the algebraic manipulation of intersecting points and lines are essentially the same, proving that we can sometimes use point and lines interchangeably.

1.3 Really Hard Calculus

BY An, Boyan, and Felix, taught by Kye

Calculus. It is hard. Quite difficult. One might even say complex. But certainly an integral part of math. Our week began with thinking about better ways to represent complex numbers, from rectangular to polar to AWA (m*trix) forms. We then turned to differentiating complex functions. It turns out, it's a lot easier to differentiate functions mapping $\mathbb{R}^2 \to \mathbb{R}^2$ than those mapping $\mathbb{C} \to \mathbb{C}$. However, this gives us derivatives for functions that shouldn't be complex differentiable, or *Felixable*. To check if functions are Felixable, we use Felix's Excellent Equation Tests (FEET), which check if the AWA form of the $\mathbb{R}^2 \to \mathbb{R}^2$ derivative satisfies some properties.

One might wonder: "Do FEET satisfy Daddie?" It turns out, yes! Anything that satisfies FEET will always satisfy Daddie (unless the function is constant). Additionally, Felixable functions preserve angles between lines; two lines will have the same angle at their intersection before and after being transformed.

After being delighted by this marvelous fact, we turned to integrating complex functions. Since our functions go from $\mathbb{R}^2$ to $\mathbb{R}^2$, we need guiding paths to tell us where to integrate. For some functions, no matter what path we took, we ended up with same result. Other functions, such as $1/z$, were mysteriously fickle and unpredictable, changing with our chosen path of integration.

Interestingly, functions that were Felixable at all points had the same integral regardless of the curve chosen. Using this, we calculated an integral for all functions of the form $\frac{f(z)}{z-k}$ and showed that we could also find the derivative of an arbitrary function by integrating it.