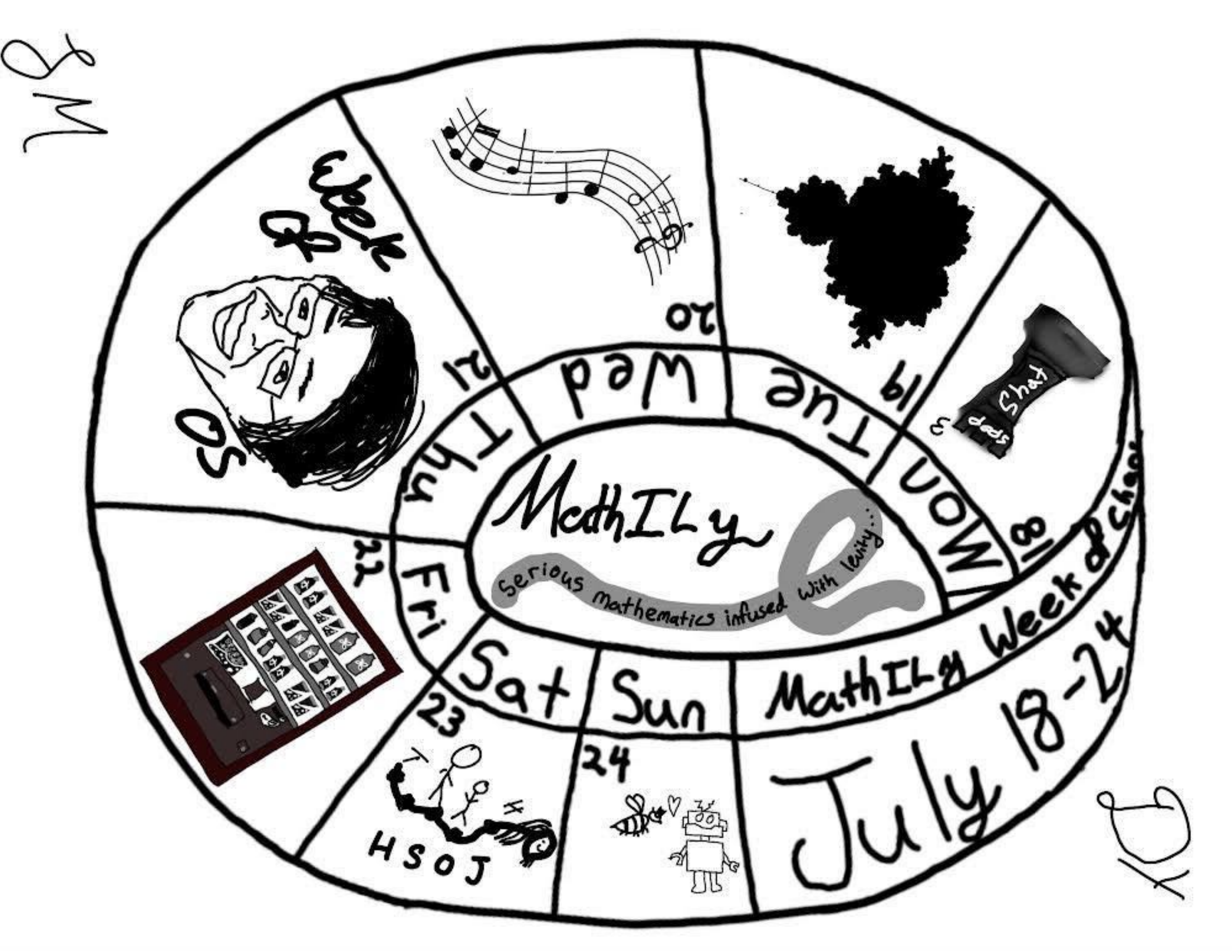
MathILy
serious mathematics infused with levity...
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MathILy Week of Chaos
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the MathILy Record of Mathematics (RoM)

Issue 3: July 25, 2022

OAT RUSH: Ana W Toh, Andrew G, Andrew Z, Annie E, Annie W, Bennie Ram Vauhf, Brandon, Caoilainn, Celene, Charles, David, Dinah Thather, Fahran, Grant, Hana, Heidi, Isaac, Jack B, Jack C, James, Jenni, Jonathan, Josiah, Kira, Lijia, Matthew, Natasha, Neil, Noah Ean, Olivia, Owen, Rahul, Rebecca, Rosalba, Tara of Hunh, Vincent, Zach, Zeke

STEROID: Timi, Jack B, Kiran

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In this issue...

2.1 Alvin: Because the Internet: Information Theory BY Rahul

Which Letters Can I Remove? In the world of the internet, brevity is your friend and efficiency reigns supreme. So how can you take a phrase and "delete as many letters as you can?" We explored the different ways in which we could shorten this phrase. Maybe we could remove all the a's, or perhaps all the vowels? However, we soon realized that many of our options could change the entire meaning of our words! We decided to take a closer look at what constitutes a valid code. That is, the conditions that make a code decipherable. Enter, the binary tree: a way to generate valid codes. In particular, we focused on codes that take vowels and output binary strings. We used our knowledge of probabilities and expected values to determine when our code was optimal. That is, we identified conditions that we must follow to minimize the length of the binary strings produced by our code.

Information Inequalities With our new knowledge of optimal codes, we went on to investigate how much information a code could give us. To understand exactly what "information" is, we looked at the popular New York Times game, Wordle. We found that hints like, "the first box contains a letter" gave us information on an event that already occurs with a probability of 1, so it gives us zero information. On the other hand, the smaller the chance of an event occurring, the greater the amount of information it gives us. For instance, not many words contain the letter Q, so knowing that a word has the letter Q narrows down the possible words, giving us more information. We found a function that, amongst other properties, always passes through the point (1,0), so if the probability were 1, we would get an expected information of 0. From there, we developed and tested examples to find that the expected information of our binary tree codes were lower bounds for our expected code lengths!

Eradicating Errors Of course, when it comes to transmitting information, many things can cause interference (e.g. a car horn when you are speaking to someone, your phone's pesky autocorrect when you're texting someone, or even background radiation interfering with messages from the James Webb Telescope). As the sun set on the Week of Chaos, we determined how we could potentially find and correct such errors. Why? Because the internet.

2.9 David: Ramsey Theory BY Owen

In a strange land, the builders and painters are fighting a war; a war for a far more honorable cause than any revolution we've ever fought, and much more *colorful* than anything we could imagine. Builders build things, and painters paint things — a fact that took us approximately one day to prove. But builders also love order and painters love *chaos*… so in the first round of this war the builder tries to force the painters to make order and the painters try to avoid the doomed, monotomous, cliques.

It turns out that the builder will always win, as long as they build enough roads. But how do we determine exactly how many roads are required to force the builder to lose? We can use some really nice recurrences to find a good upper bound for the number of roads the builder needs to build to win. But what about a lower bound? How many roads can the builder build, such that we know the painter can win for sure? Erdős actually figured out this problem a long time ago, but the gap between the best-known lower bound and the best-known upper bound, both of which were strikingly close to the ones we proved, is still astronomically large. It turns out that for such a simple premise, Ramsey Theory is a field of study with many, many open questions.

Joel Spencer once said, "Erdős asks us to imagine an alien force, vastly more powerful than us, landing on Earth and demanding the value of R(5, 5) or they will destroy our planet. In that case, he claims, we should marshal all our computers and all our mathematicians and attempt to find the value. But suppose, instead, that they ask for R(6, 6). In that case, he believes, we should attempt to destroy the aliens."

It turns out that roads and colors can be applied to so much more than just the ramsey numbers. By using a little bit of clever *bijectioning*, we can color the integers and try to prevent monochromatic (x, y, z) such that $x + y = z$. Or, like David proudly announced in the final 40 minutes of class, we could use it to prove Fermat's Last Theorem… or, at least, anti-prove it in mod p for sufficiently large primes. In particular, $x^n + y^n \equiv z^n$ has a solution for any fixed n in sufficiently large prime modulus. If only we had infinite colors…

2.10 JoSho and Katie: How the PRiME FaCTORs Took Over MathILy

BY Rebecca and Fahran

The PRiME FACToRs and greater MathILy staff have a difficult problem: cake. Specifically, some staff members want to change the standard cake served for birthdays at MathILy. 5 options were suggested for the next cake to be served: same as before, Neapolitan cake, ice cream cake, teal cake (vanilla cake but colored teal), and a giant block of icing. Each staff member wrote down a ranked list of their preferences, but now the PRiME FACToRs have to count the votes. We came up with a definition for what a voting method was and then brainstormed many different methods, including only counting first-choice votes, having different ranking placements correspond to different point values and finding different cakes' total scores, and ignoring all ballots except JoSho's.

In order to decide which of our voting methods was best, we started to define different conditions that a fair voting method should adhere to (such as the Dominance Condition and the Submissive Condition) and prove whether our proposed voting systems met the conditions. For instance, the JoSho's Choice method satisfies the Uno Reverse Condition, which states that if cake A is the current winner and all ballots swapped A and B on their preferences list, cake B should now win. We made a large chart for all methods and properties to store the results of our proofs. Then, we continued by writing proofs how different properties implied or excluded each other. We showed that a voting system that always picks exactly one winner cannot treat both all candidates and all voters equally. We then defined a power ranking, where instead of choosing one winner, a voting method ranked all of the candidates in a defined order. We updated our voting methods to return power rankings instead of individual winners and redefined our properties to better fit these new voting methods.

JoSho then defined a forcing set for A over B, which is a subset of the total voters that could unilaterally cause candidate A to be ranked over B in the power ranking, regardless of other people's votes. He also defined a conspiracy as a group of voters that could force A over B for any candidates A and B. We made conjectures about forcing sets, what happened when they split into smaller groups, and how they related to conspiracies. In fact, it became clear that under certain circumstances, it was actually quite easy to form a conspiracy. Thus, we discovered exactly how the PRiME FACToRs rigged the cake vote and took over MathILy.