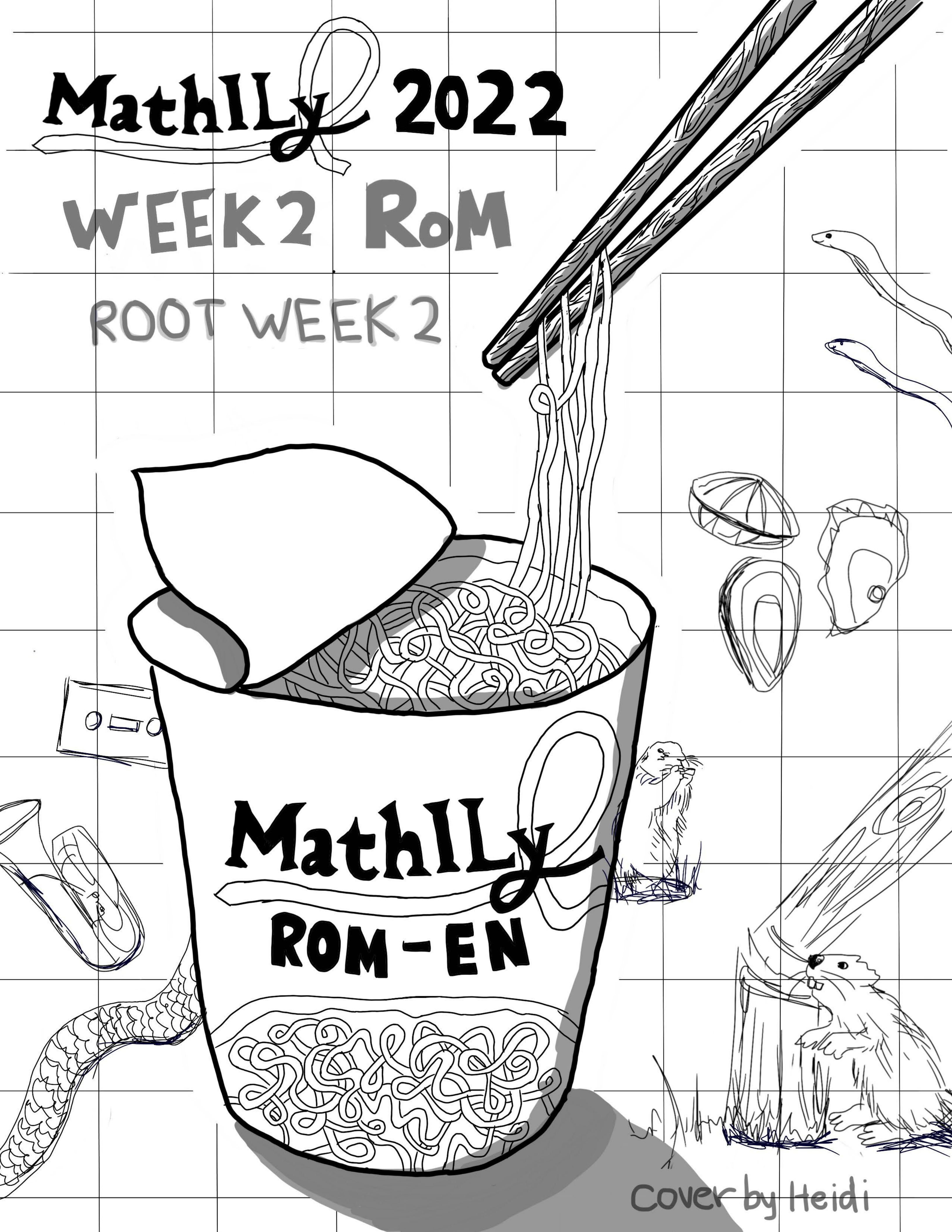
MathILy 2022
WEEK 2 RoM
ROOT WEEK 2
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ROM-EN
Cover by Heidi

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Welcome…

…to the 2nd issue of the Record of Mathematics of MathILy 2022. Check out our Root Class summaries to read about Oysters, Yoga Socks, Groundhog Sets, and many more. During our Daily Gathers we investigated Carpets, Lit Sponges, Hiding techniques, how to not summon demons, and more (check out the Daily Gather section for details!). Finally, for some entertainment, look into our Fun But Important Articles section about Zomes, an epic tale of 4 kingdoms, and some fun proof techniques.

In this issue…

3.1 Monday: Stealthy Carpets BY Eric

We began our second week at MathILy with a Daily Gather given by Daniel Studenmund examining the stealthiest of all carpets. Daniel first helped us define a triangle, which we agreed to be a region of space bounded by 3 straight line segments. It is important to note that straight is intrinsic to the surface of the figure, as demonstrated with a curved sheet of paper.

Next, to create our stealthy carpet, we taped 7 equilateral paper triangles around a fixed point, ending up with a deformed heptagon. Next, using our definition of a triangle, we drew a triangle on this carpet, such that the center of the carpet is contained within the triangle.

With our triangle, we were tasked with measuring each angle and reporting our findings to this table:

Angle 1	Angle 2	Angle 3	Sum
25	35	65	125
34	59	29	127
40	35	50	125
20	28	75	125
39	41	42	121

Not only did the angles not sum to 180°, every person's triangle seemed to sum to around 120°!

We explored the way to prove that such a triangle's angles indeed sum to 120° by examining a triangle $\triangle ABC$ and, given a center O, the new triangles $\triangle ACO$, $\triangle AOB$, and $\triangle COB$, each of which is co-planar. Each of these new triangles had properties of normal triangles we could use.

Yet, before we finished celebrating a slick solution, we were presented with another problem: What if we combined two of our carpets so they overlapped at two equilaterals? Through another round of frantic taping and drawing, we were able to create a few of these carpets, and we found that angles seemed to sum to 60°. We found this last proof was quite similar to the case of 1 carpet, but this time we would consider two centers O_1, O_2.

If we follow this pattern, what would a triangle on 3 carpets look like?

3.3 Wednesday: Lit Menger Sponge BY James

Wednesday's Daily Gather was given by Ethan Berkove. Ethan works at Lafayette College and is so far the only speaker who went to college with an acronym that is a subset of the acronym of the grad school he went to. Ethan also loves algebraic topology, and he forces his sons to help him build a 3D Menger Sponge.

What is that, you ask? Well, Ethan began with the Cantor set C, which is interesting both in its construction and its properties. The process starts with the line segment from 0 to 1. At each step of the process, the middle third of each line segment is removed, leaving smaller segments. This process is repeated infinitely, removing all possible intervals and leaves only points - "dust," as it is formally called. Ethan then defined closed and bounded sets, and he asked us about whether various rationals were in C. Although $\frac{1}{5}$ and $\frac{1}{6}$ are not in C, we saw that $\frac{1}{4}$ is. After telling us how to describe points in C as sequences of Ls and Rs, we found how to represent each point in ternary and learned more about infinite decimals.

Figure 1: The Cantor Set

Then, Ethan introduced us to $C \cdot C$, the 2-D version of the Cantor set. We proved that a light ray with the equation $y = -x + b$ always intersects a point on $C \cdot C$, so there always exist two points $x, y \subset C$ such that $x + y = z$ for any z in $[0, 2]$. We also began talking about the gaps created at each step of the Cantor approximation.

Afterwards, Ethan showed us how to make the 2-D Sierpinski carpet with 2 Cantor sets, and showed us a video - **Mathematical Impressions - The Surprising Menger Sponge Slice**. We then tested ourselves trying to recognizes various 2-D cross sections of the 3-D Menger Sponge based on which gaps could be seen.

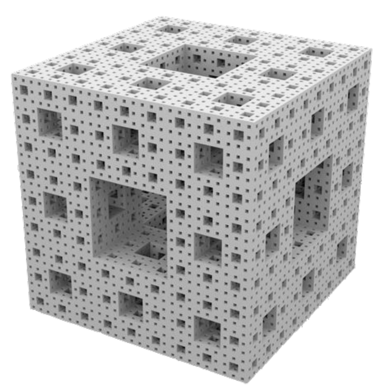

Figure 2: The Menger Sponge

Finally, Ethan talked about how light could only pass through certain gaps, and he showed how his team had calculated how much light would pass through the sponge at certain angles. As it turns out, the only good place to shine a light is perpendicular to a face of the cube or close to it - a thrilling conclusion to this sponge saga.

3.4 Thursday: Super Secret Soup or Sandwich BY Jonathan

In Thursday's daily gather lead by Nadav Kohen, we looked at pairs of functions called ciphers. Each pair has an encryption function, E, that takes a key, k, and a message, m, and outputs some cipher text, c. The second function of the pair, D, decrypts the cipher text with a key to get the original message. In other words $D(k, E(k, m)) = m$.

We brainstormed some examples (and non-examples) of ciphers, and discussed some of the useful characteristics. Farhan made the key observation that even while knowing the function used, a force of evil should not be able to gain any information about the message from the cipher text, provided they do not have the key.

This observation prompted Nadav to define a perfectly secure cipher as follows: for any cipher text $c \in C$, if a key k is randomly chosen from K, then for any two messages $m_1, m_2 \in M$,

$$\mathbb{P}(c = E(k, m_1)) = \mathbb{P}(c = E(k, m_2))$$

We then proved that for a cipher with a smaller key set than message set, $\forall$ ciphers c_1, $\exists$ a message m_1 such that $\forall$ keys k_1, $E(m_1, k_1) \neq c_1$.

Using this lemma, we were able to prove that any perfectly secure cipher must have at least as many keys as messages. Unfortunately, having such large key-sets is impractical. In the real world, there are far too many messages to have that number of keys- the keys would simply be too large!

So, Nadav introduced a way to make a cipher "secure enough". This definition relied on a game between a the encryption machine and some force of evil defined as such:

1. The forces of evil give the machine two different messages, m_1 and m_2.
2. The machine randomly picks a game, where one game encrypts m_1 and the other encrypts m_2 with a randomly chosen key
3. The evil forces then guess which game was played.

We then consider the positive difference between the probability that evil forces guess game 1 when game 1 was used and the probability that evil forces guess game 1 when game 2 is played. If this difference is a very small positive ε, we say the cipher is "secure enough".

Finally, we considered a different game in which the encryption machine picks a random message out of the message set and a random key and outputs the encryption. The evil forces then guess which message was encrypted. We define the evil forces' advantage to be the difference between the probability that they guess correctly and the probability that random guessing gives the right message.

By combining these two games, we were able to show that security with respect to the secure enough game implies security with respect to the message recovery game.