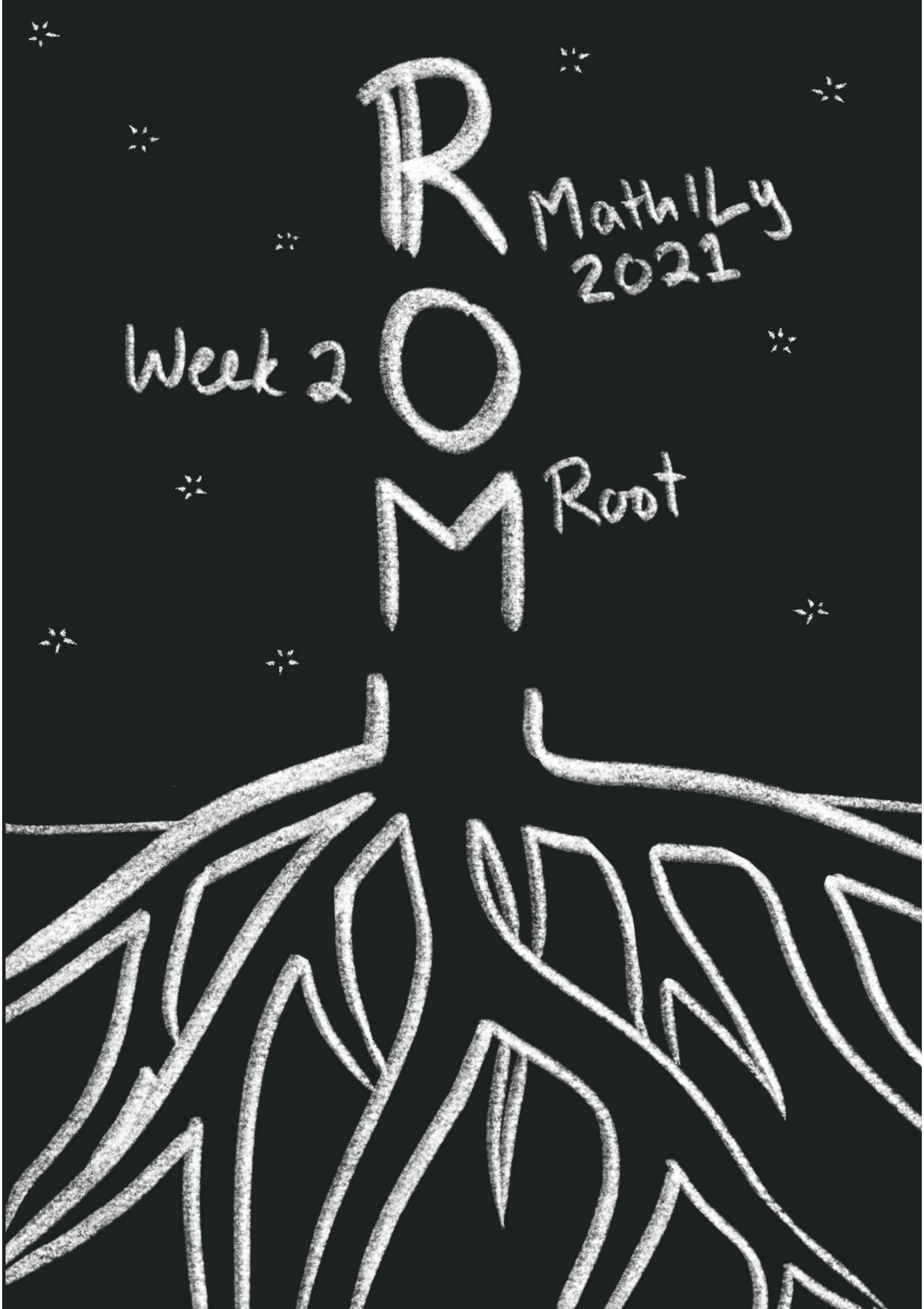
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Week 2
Root

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Welcome...

...to the second issue of the 2021 RoM! In our final week of Root class, we explored pig pigeons, problem-set eating dingos, baby exchanges, and more (details can be found in Root Class Summaries)! In this week's Daily Gathers, we helped spiders cut cheese, tossed Sammy the Symmetrosaur around in tetrahedrons, explored changing the order of selbairav, got nightmares from blinking donuts, and spread a SCANDALOUS rumor (see Daily Gather Summaries). Finally, visit Fun Stuff to check out various quotes and learn what awaits you if you dare miss the RoM meeting!

In this issue...

3.3 Wednesday: Connor and Symmetric Polynomials BY Perryn

Connor Ahlbach is currently part of Texas State University. For his undergrad, he went to Harvey Mudd, which is the same college as one of our current AIs. He is in the process of writing his own linear algebra textbook!

A Pat Polynomial is a polynomial where the order of the variables doesn't matter. Since the order of the variables doesn't matter, tPa sometimes likes to jumble up the letters of his own name! Another restriction of a atP polynomial is that the degree of the polynomial must be at most the number of variables. This means that

$$P(x_1, x_2) = x_1^5 x_2^3 + x_1^3 x_2^5$$

is not a Pat polynomial.

Here is a question that one might have:

Question: What is the minimum set of generators of Pat polynomials of degree n? The possible operations are adding two polynomials and multiplying by a scalar.

Let's see: We can experiment by writing out the taP Polynomials of degree 1, 2, and 3.

Degree 1	x_1
Degree 2	$x_1^2 + x_2^2, x_1x_2$
Degree 3	$x_1^3 + x_2^3 + x_3^3, x_1^2x_2 + x_2^2x_1 + x_1^2x_3 + x_3^2x_1 + x_2^2x_3 + x_3^2x_2, x_1x_2x_3$

Oof, that last one is a bit ugly. What we can do is suppose that $\rho = x_1^{\lambda_1} x_2^{\lambda_2} \dots x_k^{\lambda_k}$ shows up in a aPt polynomial. Then, in order for it to be part of a Pat polynomial, all $x_{i_1}^{\lambda_1} x_{i_2}^{\lambda_2} \dots x_{i_k}^{\lambda_k}$ must appear, where

$$\deg(\rho) = \sum \lambda_i = n$$

and

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k.$$

(This is just to satisfy the definition of a Pat polynomial).

As a side note, we know that the number of unordered partitions of n is asymptotically

$$\frac{1}{4\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}}.$$

Now the question is, what if you allow for multiplication of generators?

Interestingly, it actually turns out that one can form all Pat polynomials just by using the n polynomials

$$(x_1 + \dots + x_n), (x_1^2 + \dots + x_n^2), \dots, (x_1^n + \dots + x_n^n).$$

3.4 Some Food For Thought: BY The Editors