THE record OF Mathematics

ROOT WEEK 1, 2021

the MathILy Record of Mathematics (RoM)

Issue 1: July 4, 2021

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Welcome...

...to the first issue of the 2021 RoM! We had an exciting first week of *Root class* filled with muffins, spiderwebs, anthills, and dumplings (check out the Root Class Summaries for details!). In this week's Daily Gathers, we made frogs engage in cannibalism, learned of Josh's limited vocabulary, had a special guest from UMass Amherst speak, and much more! (See the Daily Gather Summaries for more!) Be sure to check out the various quotes submitted in Fun Stuff as well! Finally, take some time to admire Jonathan's artwork and Giacomo's haiku on page 23. Enjoy!

In this issue...

please read this RoM we spent a lot of time on it ty :)

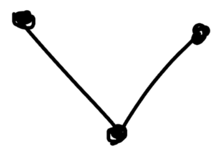
3.2 Monday: The Umlaut Snäkes World Tour! BY Alyna

Cool Things About Corrine! Corrine was PRiME at MathILy in 2014 and became an AI ever since. She attended Sarah Lawrence College, where she did theatre and math. She wrote a play that is performed across the country and she loves combinatorics!

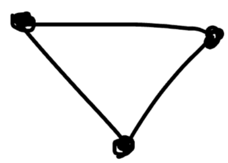
Here Come...*drumrolls*...The Umlaut Snäkes! The Umlaut Snäkes are a world-renowned heavy-metal snake band that play electric guitars. This year, the Umlaut Snäkes are coming to MathILy as a part of their world tour! However, they need help with connecting their amps.

The amps are connected with each other using wires. The reason why we want to connect them is because the amps can amplify each other and create greater volumes.

The biological structure of snakes' brains and auditory system enables them to stand high volumes, so most amp setup are fine; however, they have an upper limit, so some setup are not. Here is an example of an okay amp connection (we will represent the amps by dots and the wires by lines):



Here is an example of a not-okay amp connection:



This connection is not okay because it creates a Triangle of Doom (ToD)! Fortunately, ToD is the only type of amp connection that the snakes are not okay with. Therefore, any amp setup with no ToD is an okay setup.

The Problem Given a set of amps, what is the maximum number of wires that can be added without getting any ToD?

The Solution To approach this problem, it would be best to start making a table with 4, 5, and 6 amps! This is what we got:

Number of Amps	Maximum Number of Wires
4	4
5	6
6	9
⋮	⋮

Connor conjectured that the maximum number of wires for a setup of n amps is $\lfloor \frac{n^2}{4} \rfloor$. Try proving this conjecture using induction!

3.3 Tuesday: Sums of squares about triangles BY Madeleine

Looking at points in the plane of the form (# of strands, # of triangles), can we create a graph with those properties? If so, we can plot the point. Looking at their distribution, it seems that they mainly lie between two parabola-shaped curves.

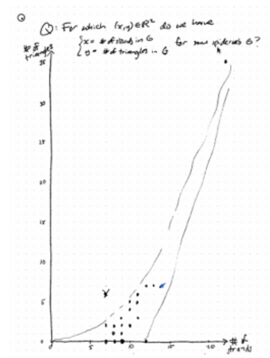


Figure 4: (# of strands, # of triangles) found in graphs with 7 vertices

This becomes harder to plot as the number of vertices increases. To make it easier to plot and visualize, we can think of densities in order to normalize. In particular, we can set

$$x = \frac{2 \cdot \text{\# of strands}}{(\text{\# of vertices})^2}, \qquad y = \frac{6 \cdot \text{\# of strands}}{(\text{\# of vertices})^2}.$$

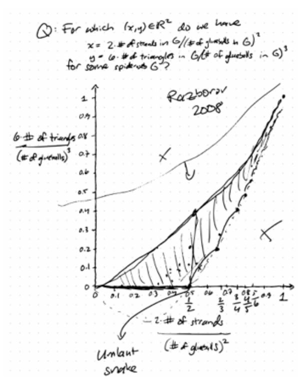


Figure 5: Normalized plot of the amount of triangles in graphs

We can notice that we can't reach everything between the two curves. In particular, the scalloped edge on the bottom parabola originates from the formulas we proved during Corrine's Umlaut Snake Daily Gather. This model was found to be correct in 2008 whereas the the Umlaut Snake formula was discovered in 1907.

Along with looking at triangles, we can also look at other shapes, trying to find relationships between graphs and sub graphs. One practical application of this can be found in the study of the brain. Considering neurons and their connections as a graph, we can try to deduce functionalities based on the distribution of smaller sub-graphs (looking at the whole thing can be too hard because of the sheer number of neurons in a brain).

There are different ways of counting the number of times a sub-graph appears in a larger graph. For example do you count it as a sub-graph even if some of the vertices are also connected in different ways? Can you send two vertices to the same point? We can define three methods of counting: the exclusive (answer to the questions: no, no), the relaxed (yes, no), and the free way (yes, yes).

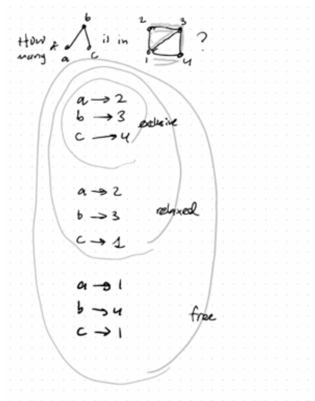


Figure 6: The three counting methods on a particular example

In the example in Figure 3, after dividing by all possible maps, we find that exclusive gives $\frac{4}{24}$, relaxed gives $\frac{16}{24}$, and free gives $\frac{26}{64}$. In fact, as the number of vertices approaches infinity, relaxed and free give the same numbers. Generally, we use the free method. We can define $t(H, G)$ as the number of sub-graphs H within G, the main graph. The general goal is to develop algebra (multiplication, averaging...), involving these quantities in order to prove inequalities for the kinds of graph that we saw in the beginning.

We then defined and proved certain operations with these quantities such as multiplication, partially labeling graphs, a "gluing" method, and symmetrization.

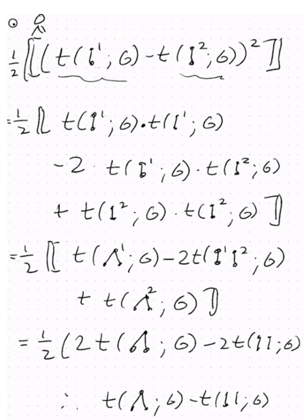


Figure 7: Proof of inequality

Annie Raymond left us with the proof of an inequality (shown above), and a problem to attempt our leisure. The proof of the latter gives us the space between the curves from Figure 2.