down in the classroom, the front of the room has the board and the left side has the air conditioner heating up the room from the -50°C temperature outside. Understandably and deservedly for all their hard work on the real, totally non-rote, math of MathIGy, the lower-numbered students get the first pick of seats. So, when the seating configuration is complete each row increases to the right and each column going down.

The one form of entertainment the students have, aside from playing in the snow in the -60°C mornings, is to play a game with their seats. In this game, a number of the seats are removed from the top/left room, all the students sit in a proper configuration, and then chairs are put back one by one, moving only from bottom-to-top / right-to-left, and students can fill into adjacent seats to improve their position, with higher-numbered students yielding to lower-numbered students.

The MathIGy students, so accustomed to their rote memorization, could not see this game for what it was to us - an interesting puzzle. We claimed that the order in which desks were replaced does not affect the final position, and were able to prove that the relative position of n and $n+1$ stays the same based on the extension of the proof for 1 and 2. Connor claimed if we add some fancy nonsense to this idea we could prove the claim.

Back at MathIGy, two classes were being combined into one, with one class having all worse rankings than the first and this ended up farther from the heating and board. However, the Coordinator of Orderly Mathematics and Predictor of Oppressive Snowstorms in the Tundra Environment (COMPOS-iTE) was talking with the instructors and they realized they had their ranking system all wrong. The people in the top class needed to be swapped with those in the bottom class, while keeping the internal class rankings the same. We considered two ways to do this, one by swapping out the worst people in the previously better class first and the other by swapping out the better people in the now better class first. We could see that the two methods seemed to have the same result, and with induction we can extend a small argument to show this.

Suffice it to say, Connor's Daily Gather gave us a better understanding of why we should be interested in MathIGy and work hard at our boredom-inspiring factoring and rearranging of logarithms to ensure we don't freeze in the Siberian winter.

2.3 Wednesday: Shamil—Distribution of wealth among the lines

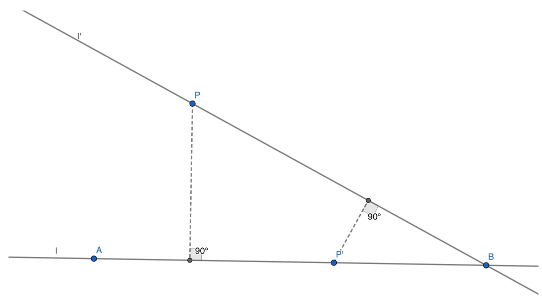
by Youbin Cho

Shamil started us off with a problem: Given $n \geq 3$ on the plane $\mathbb{R}^2$, not all collinear, can we find a line which passes through exactly two of the n points?

We let S be the set of n points. When we couldn't figure it out, Shamil gave us a hint: Consider the set TS of all pairs (P, L), where $P \in S$, $P \notin L$, and L is a legit line (contains at least 2 points of S). The set TS is non-empty because not all points in S are collinear. So, $TS \neq \emptyset$, and TS is a finite set (as there are only finitely many legit lines).

When we still could not find a solution to the problem, Shamil gave us another hint: Consider the function $f : TS \to \mathbb{R}^2$, where (P,L) $\mapsto$ distance between the point P and the line L. Using this idea, Alexander and Anthony proposed a solution to the problem:

Take a pair (P, ℓ) that minimizes the function, and assume for the sake of contradiction that line ℓ contains 3 points: A, P', and B. When a perpendicular line h is dropped from point P to line ℓ, h divides line ℓ, and so there are at least two points on the same side. Say P' and B are on the same side. Then, let the line containing P and B be called ℓ'. As P' and B are on the same side of line ℓ , $f(P', \ell') < f(P, \ell)$. Contradiction!



After this, Shamil proposed another problem: Given n points on the plane, not all collinear, how few lines do these points determine? Will used induction to prove that the minimum number of lines is n. At the end of the daily gather, Shamil proposed a final question. If a k-rich line is a line that passes through exactly k points, are there more than one 2-rich line? It turns out that the number of 2-rich lines is at least $\left\lfloor \frac{n}{2} \right\rfloor$ for n large enough according to a recent result of Ben Green and Terrence Tao.

2.4 Thursday: Nate Harman (U. Chicago)—How to Win at Tangrams

by Joeli Leyendecker

Thursday's Daily Gather focused on a toy that many of us had seen before: tangrams. For those who don't know what tangrams are, they're made by using straight cuts to cut up a square of paper. Once the square is cut up, see what shapes can be made by putting the pieces together such that every piece is used and no pieces overlap. Nate passed out paper and scissors and told us to find what sorts of shapes could be made using these rules. We rapidly found a way to make a square (to quote Dimitri, "I take a square... and that's it.") and a $2 \times (1/2)$ rectangle. Nate then gave us a few more challenges: could we make more rectangles? Parallelograms? Equilateral triangles?

Daniel found a method of making parallelograms by cutting diagonally from the corner of the square and moving the newly-cut piece to the other side of the square. By repeatedly doing this, he could make parallelograms of indefinite thinness. This parallelogram-making method led Will to a method of making any rectangle of size $a \times (1/a)$. Winston then found a method of making all triangles of area 1.

Nate gave us a final question; what can we make from a square? Shortly before the end of the Daily Gather, we showed that all polygons of the same area as the square can be triangulated (and thus made from tangrams). Nate told us that we proved a theorem, the name of which was given very quietly while other people were talking. I wrote it down as "Boluai-Dashuin (?¿?) Theorem," but the internet leads me to believe it's actually called the Wallace-Bolyai-Gerwien Theorem. After telling us that, sadly, this property doesn't hold in 3d, Nate ended the Daily Gather and sent us to dinner (and let us keep our tangrams!).

(He did take away the scissors, though.)

2.5 Friday: Rosemary Guzman (U. Illinois, U. Chicago)—Almost a Tessellation Celebration (but not really)

by Pico

MathILy started by considering a "motivating question", one that we may or may not figure out the answer to. It was "what triangles tessellate". The question was supposed to be open ended.

To figure out what this question meant, we had to consider what triangles even exist? Does a triangle with angles equal to $\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}$ even exist? Surprisingly, this triangle can exist on a sphere. We then considered what a line between points p and q is. On a plane, it is the shortest path from p to q. What could it be on a sphere? It turns out to be the great circle with p and q.

We considered if for given $p, q, r \in \mathbb{N}$, the triangle with angles $\frac{\pi}{p}, \frac{\pi}{q}, \frac{\pi}{r}$ exists. It is known that if $\frac{1}{p}, \frac{1}{q}, \frac{1}{r} = 1$ then it forms a triangle on the plane. It was shown that if $\frac{1}{p}, \frac{1}{q}, \frac{1}{r} > 1$ then it forms a triangle on the sphere.

Then, Rosemary inquired us to find out about the area of a triangle on a sphere (with surface area 4π) with angles $\frac{\pi}{p}, \frac{\pi}{q}, \frac{\pi}{r}$. We showed that the area is equal to $\frac{\pi}{p} + \frac{\pi}{q} + \frac{\pi}{r} - \pi$.

To find out why the angles uniquely determine the area of the triangle, we considered solutions to $\frac{1}{p}, \frac{1}{q}, \frac{1}{r} = 1$ and $\frac{1}{p}, \frac{1}{q}, \frac{1}{r} < 1$. The first has three integer solutions which are $(3, 3, 3), (4, 2, 2), (6, 3, 2)$. On the other hand, there are lots of solutions to the second, for example $(7, 3, 2), (8, 3, 2), (9, 3, 2) \ldots$

By the end of this daily gather, we discovered more about triangles considered impossible in the Cartesian plane and went to the dinner with our balloons.