Contents

Week of Chaos

Math Saves the World: Combinatorial Optimization

Zoë and Leslie

The process of optimization involves the following steps: identify the problem, collect data, and build a mathematical structure that reflects the essential aspects of the problem. We covered true stories that exemplify how combinatorial optimization is used in the real world. One example was about implementing positive bag match in airports to help prevent terrorist attacks. Positive bag match means that the airline makes sure that no one checks a bag onto a plane without getting on the plane themselves. Then, we came up with our own examples of problems that could be solved using combinatorial optimization, like ceiling light placement and traffic light timing. We also applied the steps of optimization to our own problem of assigning people days to take notes on.

Geometric Classification

Jon and Romain

In this class we studied ways to parameterize geometric objects and the properties of these parameterizations. We began by using typical triangle congruence theorems (SSS, SAS, and ASA) to parameterize labeled triangles. The boundaries of our parameter spaces contained interesting degenerate cases, some of which had different dimensions in different parameterizations. We used the Law of Sines and Law of Cosines to create maps between these parameterizations. We then switched to classifying lattices created by parallelograms in the complex plane.

Moar Combinatorics

Lavinia

In Moar Combinatorics we covered Stirling numbers, falling and rising factorials, a checkers problem, and a paper-coloring problem. We also found many connections between the different topics. A Stirling number of the second kind is the number of ways to arrange a set into nonempty subsets. A Stirling number of the first kind is the number of ways to arrange a set into nonempty cycles. We found two recursions to calculate Stirling numbers.

Big Finite Diff

Alvin

For a given sequence $f : \mathbb{N} \to \mathbb{R}$, we define the first finite difference of the sequence f to be Δf such that $\Delta f(x) = f(x+1) - f(x)$. This process of taking the difference of consecutive terms to create a new sequence continues with the 2nd finite difference being $\Delta^2 f(x) = \Delta f(x+1) - \Delta f(x)$ and more generally, the mth finite difference of f is defined recursively as the sequence $\Delta^m f(x) = \Delta^{m-1} f(x+1) - \Delta^{m-1} f(x)$. If there was a finite difference $\Delta^m f$ of a given sequence f that became a constant sequence, we were able to express the x^{th} term in a sequence, $f(x)$, in terms of its finite differences. This led us to finding and proving a general formula for all functions whose finite differences eventually became constant: $f(x) = \sum_{j=0}^{m} \binom{n}{j} \Delta^j(0)$.

How to Treat TMM Syndrome

Zoë

In class, we discussed the devastating effects of Too Much Math syndrome. After studying too much math, Alex wandered mindlessly between the buildings on campus. We chose probabilities for Alex to travel from each building to each other building, and entered them into a matrix. We multiplied this matrix on the left of a vector that represented where Alex started. Applying the probability matrix n times gives the probability that Alex will be in each location in n hours. We discovered that as n increases, the entries in the matrix reach a limit. Then we considered the scenario where Alex's movements between Park, Denbigh, and Erdman were driven by hidden emotional states. In this example, the observer can only see where Alex is going, but not what his emotional state is. We asked, but did not answer, if we could figure out his mental state based on our observations of his location.

Projective Geometry

Khunpob

How do you describe perspective as used by our eyes? Why is it that lines that are parallel in three dimensions can intersect in a two-dimensional image? We started this discussion by defining the projective plane, $\mathbb{RP}^2$, which consists of planes and lines through the origin projected to lines and points on the $z = 1$ plane, along with a line at infinity. Lastly, we covered transformations on lines and conic sections, and how linear transformations affect the projections on the projective plane.

The Algebra Safari

Abhi

In Algebra Safari, we discussed Rinds. We looked at the relationships between two Rinds, Z_n and Z_m. While studying these two Rinds, we defined a homomorphic function and created generalized conditions for when a homomorphism exists from Z_n to Z_m. We found all the distinct homomorphisms between Z_n and Z_m, and we found a formula for the number of surjective homomorphisms from Z_n to Z_m.We found another way to tell if a function is not surjective by looking at $gcd(n, m)$. We also looked at bijections between other Rinds, such as the flips and rotations of a equilateral triangle.

Surreal Numbers: A Play in Five Acts

Catherine

John Conway created the surreal numbers with two rules:

1. x is a surreal number iff $x = \{X_L | X_R\}$ where X_L and X_R are sets such that $x_l \ngeq x_r$ $\forall\ x_l \in X_L$ and $\forall\ x_r \in X_R$.
2. Given $x = \{X_L | X_R\}$ and $y = \{Y_L | Y_R\}$, $x \leq y$ iff $x_l \ngeq y\ \forall\ x_l \in X_L$ and $y_r \nleq y\ \forall$ $y_r \in Y_R$)

He also defined the numbers $-1, 0, 1$. We were tasked with creating more numbers and proving fundamental operations such as addition, transitivity, and associativity. We also needed to prove that $\ngeq$ was equivalent to $<$. Combining concepts we knew about binary and infinity, we were able to describe fractions and infinitely many infinities using left and right sets.

Knot Theory, Practice

Romain

How can one tell whether two knots are the same? In order to answer this question, we began by learning how to draw knot projections, and then we analyzed how one can manipulate the projection while preserving the knot it represents. We also investigated different types of knots, such as the torus knot and the braid knots, which turned out to have an underlying group structure and some very weird algebra rules. Though manipulating projections of all of these knots allowed us to provide conclusions about when two knots are the same, it could knot prove that two knots were different. As a result, we began investigating knot invariants, hoping to find a way to truly differentiate knots. We were therefore introduced to the FLYPMOTH polynomial, which is an invariant two variable polynomial (and is really quite hard to compute). It allowed us to answer the question which motivated the class: the two trefoil knots are indeed different.

Groups and Graphs

Milan and Catherine

We began Groups and Graphs by discussing the mantra of group theory: "Every group is a collection of symmetries of some object". We then examined various of groups and created objects of which the groups could be symmetries. For example, the Klein 4-group, with four elements that are their own inverses, acts as the symmetries of a non-square rectangle, while $\mathbb{Z}$ is the translations of the real number line. We defined a symmetry of an object X as a bijection $\phi : X \to X$ that preserves the structure of X (e.g., if X is a graph, vertex adjacency is preserved), then wrote the set of symmetries of X as $Sym(X)$. A group G "acts on X" if there exists a function, also called an action, $f : G \to Sym(X)$ that preserves identity and such that $f(g_1 g_2) = f(g_1) \circ f(g_2)$ for all $g_1, g_2 \in G$. So, the action of G on X is a homomorphism. If this homomorphism is injective, then the group action is faithful. We defined notation for group action as $g \cdot x = [f(g)](x)$ for $g \in G$, $x \in X$. Then, we wanted to find for any arbitrary group G an object X that is acted on by G. This can be done by constructing a graph Γ in which the vertex set of Γ is labeled by the elements of G, and there is a directed edge from vertex g_1 to vertex g_2 labeled by a generator s_i if and only if $g_1 * s_i = g_2$, where $*$ is the group operation for G, for all $g_1, g_2 \in G$ and for all s_i. The generators $s_1, \ldots, s_n$ of G are elements in G such that every element of G can be written as a product of the generators and their inverses. It can be shown that the action of G on Γ is defined by $g \cdot h = g * h$, for all $g \in G$ and for all vertices h of Γ. This Γ is called the Cayley graph for G. On the last day, we discussed relations and relators of groups: non-trivial collections of elements which combine to the identity. The infinite group with no relators is called the free group, and its Cayley graph is a self-similar shape with countable vertices, but uncountable endpoints.

A Serious Study of a Pastime

Wen

In Serious Study of a Pastime, we looked at games. Specifically, we looked at a game called Nim. In Nim, you aim to remove block towers and force your opponent to have no moves. We investigated Nim in many ways, such as assigning values to games, and finding an optimal strategy. We also investigated other games, such as Chop and Chomp. We showed that any game of Chop is equivalent to a game of Nim, and we suspected the same of Chomp. We also proved many theorems about a certain type of game called impartial games. These are games where both players have the same possible moves to take, and which player you are doesn't change what moves you can take. Some examples are Nim, Chop, and Chomp. One theorem we proved if that if two games are equal, then a possible move in the first game is equivalent to a possible move in the second game. By building up many relations between such games, we ultimately were able to prove that all impartial game are equivalent to Nim.

Ramen Theory

Artur

Everyone remember the coloring-an-edge game in Root class? We started the week with something similar: coloring edges of a complete graph. We analyzed the following question: with two colors, how many nodes are needed in order to always have at least one triangle whose edges are all of the same color? And how many are needed in order to form a triangle with one of the colors or a complete graph on four vertices with the other color?

After answering these questions, we started proving some general conjectures and looking at other problems similar to this one, like doing the same thing with more colors, with a hypergraph, or with a sequence of numbers in order to find a monochromatic arithmetic progression. We proved some theorems such as the Erdős-Szekeres Theorem and Schur's Theorem. We managed to see that it's possible to find order or patterns even in chaos.

Le Baguette Theory

Brian

The integral of a function f can normally be interpreted as the area under a curve between a and b, but when a characteristic function $\chi_A : \mathbb{R} \to \mathbb{R}$ for a fixed $A \subseteq \mathbb{R}$ given by

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

is integrated, the meaning of area under the curve no longer holds, and the traditional Riemann integral does not exist. In order to combat this problem, the Lebesgue integral partitions the y-axis. Before coming up with any numeric calculations, we had to first define the length of a given set on the real line. In order to interpret length of a set, we looked at measure theory and also defined non-measurable sets.

To define the Lebesgue integral of a function f on a measurable set A, we first choose a sequence of points $P = \{l = y_1 < y_2 < \cdots < y_n = u\}$ where $[l, u]$ is the range of f. We also define the set $A_i = \{x \in A : f(x) \in [y_{i-1}, y_i]\}$. The Lebesgue sum is then

$$L(f, P) = \sum_{i=1}^{n-1} y_i^* m(A_i)$$

where y_i^* is in $[y_{i-1}, y_i]$. The function is then Lebesgue integrable on A if $\lim_{||p|| \to 0} L(f, p)$ exists, and it is denoted as $\int_A f dm$.

Eigenonion

Cam

In Eigenonion, we looked at a subset of linear transformations of the form $Mv = \lambda\vec{v}$ (i.e., linear transformations that scale vectors by λ). λ is called an *eigenvalue*, $\vec{v}$ is called an *eigenvector*, and the set of all v satisfying the equation is called an *eigenspace*. We started with two dimensions and found matrices with one and two one-dimensional eigenspaces, two-dimensional eigenspaces, and no eigenspace. We then looked at how to find eigenvalues of a 2×2 matrix using determinants and linear algebra. We wanted to extend this to higher dimensional matrices. We proved that all matrices satisfy a polynomial equation, $a_0 + a_1 M + a_2 M^2 + \cdots + a_n M^n = 0$ and that the smallest degree polynomial which will satisfy this equation has roots that are the eigenvalues of that matrix. To conclude the class, we showed that every square matrix has a complex eigenvector.

Generatingfunctionology

Alex

A generating function generates the coefficients for a given sequence. For example, the generating function for the sequence $(1, 1, 1, \ldots)$ would be the following:

$$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$$

We examined the properties of generating functions and used them to help us do combinatorics problems easily with algebra. Generating functions are used to solve problems about distribution or recurrence, such as finding a closed form for the Fibonacci sequence. We also examined the properties of exponential generating functions, such as the following:

$$\sum_{k=0}^{\infty} \frac{n^k}{k!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = e^x$$

Exponential generating functions unleash the power of e^x in order to solve combinatorics problems in which order matters. We also examined the snake oil method, in which a combinatorial sum is turned into a generating function, and then the sums are switched. There does not seem to have any reason to work, but usually it does, so we just hope for the best.

Erdős Magic

Hector

Erdős magic is a probabilistic method to compute a restrictive condition for a claim to be true. The general structure of the proof is as follows:

1. Define a specific negative event.
2. Calculate its probability of occurring in a random instance of the situation being considered (for example, a random 2-coloring of a complete graph).
3. Multiply Pr[negative event] by the total number of distinct instances of the situation.
4. If the result is less than 1, we know that there must be a case where negative events don't exist, which is what we want.
5. This allows us to define our condition: the claim holds if the expression for the probability of any negative event occurring is less than 1.

The core of this method lies in switching the focus of the proof from direct proof of a specific case to proof of existence by probability and expected value. We can solve many problems that require us to "fill in" some part of the claim (like proving the existence of an optimal strategy for a game given some condition) using Erdős magic.

How to Play Poker

Shalin

We began by considering a simple version of poker in which both players rolled a 6-sided die as their hand while both anted $1 to start. In addition, only player 1 was allowed to bet and player 2 could call or fold in turn. We saw that a strategy that made player 1 money was one where they bet on a 1, 5, or 6. Meanwhile, player 2 would call on 2, 3, 4, 5, or 6. In this case player 1 made a net of $\frac{2}{36}$ dollars. We eventually found that a Nash Equilibrium exists for this simple game of poker in which player 1 bets on 5, 6, and also on 1 with a probability of $\frac{2}{3}$. Player 2, on the other hand, would call on 4, 5, 6, or 3 with a probability of $\frac{1}{3}$. Later, this game was expanded so that the rolls consisted of choosing a random number in $[0, 1]$ to be a player's hand. This was considered to be a decent representation of a hand in poker as there are millions of different hands that exist. In this case, we were able to see that the Nash Equilibrium exists when player 1 bets with a hand less than $\frac{1}{10}$ and when they also bet with a hand greater than $\frac{7}{10}$. Meanwhile, player 2 calls if their hand is greater than $\frac{4}{10}$. We then went on to analyze games with more rounds of betting as well as those with different bet sizes. The general method that we used to do this was using indifference equations that described the situation where a player has reach a local maximum in their expected value. This allowed us to observe the Nash Equilibrium and was a powerful method we used throughout the class.

Loops on Loops on Loops

Henry

Two loops are homotopic if they can be can be smoothly deformed into one another while remaining in the same space. In Loops, we learned that all the loops in a space can be partitioned into equivalence classes under homotopy, i.e. sets of loops that are all homotopic to one another. Different spaces have different sets of equivalence classes with various interesting characteristics. Two spaces we focused on were an infinite spiral staircase in the Icelandic Hilbert Hotel and the circular shadow left on the lobby floor by a guest moving up and down that staircase. It turns out that these two spaces have sets of loop equivalence classes that are SproGh isomorphic to the integers.

p-sychic p-henomena

Eric

We began by looking at some important properties of our number system and of absolute values. From a new definition of absolute value with similar properties, we developed a new system, called the p-adics. Because operations function differently with the p-adics than in the standard system, we then worked on showing how non-integers would be expressed. We demonstrated when such numbers could be written and how to construct their representations when possible. In this process, we also touched on the properties of polynomials in the p-adic system.

More Marxist Mayhem

Jon

We studied a new type of graph problem: when Baby Dietmar points to a node, that node passes candy to each adjacent node and loses the amount that it passes. This can be modeled with a matrix similar to the adjacency matrix made by subtracting the score of each node i from the i^{th} entry on the diagonal of the matrix. We found that the only input vectors to this new matrix that preserved the number of candy at each nest were ones that pointed to each connected component the same number of times. Then, we investigated when a graph can be brought out of serfdom, i.e. pointed to until each nest has a nonnegative amount of candy. One condition was that the sum of the candy in all of the nests, the Gross Domestic Candy (GDC), must be nonnegative because pointing preserves GDC. We discovered that for K_3 serfdom can always be ended if GDC> 0 and in one of three cases if GDC$= 0$. We also showed that serfdom can always be ended in trees and that the number of candy configurations of a graph, up to pointing, is finite.

The Characters of Groups

Cindy

We found some examples of finite groups, such as $(\mathbb{Z}_n, +)$, the symmetries of a regular n-gon, and the permutation group of $\{1, 2, \dots, n\}$, which is the group of bijections from $\{1, 2, \dots, n\}$ to itself and is called S_n. Then we learned many definitions. A *representation* of a group G is a homomorphism from G to $GL_n(\mathbb{C})$, where $GL_n(\mathbb{C})$ is the set of all invertible $n \times n$ matrices with entries in $\mathbb{C}$. Two representations R_1 and R_2 are isomorphic if there exists a matrix A such that $R_1(g) = AR_2(g)A^{-1}$. The *character* of a representation $\rho : G \to GL_n(\mathbb{C})$ is the function $\chi : G \to \mathbb{C}$ that maps each $g \in G$ to the trace of $\rho(g)$. The trace of an $n \times n$ matrix A is $\sum_{i=1}^{n} A_{ii}$, or the sum of entries in the diagonal. The *inner product* of two characters χ and ψ is $\langle \chi, \psi \rangle = \frac{1}{|G|} \sum_{g \in G} \chi(g) \overline{\psi}(g)$. We made tables of characters and inner products and proved that orthogonal characters are linearly independent. We also discussed the direct sum of representations $R_1 \oplus R_2$, which is a matrix where the top left corner is R_1, the bottom right is R_2, and the remaining entries are zeros. A representation is *decomposable* if it is isomorphic to a direct sum of representations. A subspace $V \subset \mathbb{C}$ is invariant if for all $v \in V$, $R(g)v \in V$. We proved that if B is a homomorphism $R_1 \to R_2$ and a matrix such that $BR_1(g) = R_2(g)B$, then the kernel of B is invariant under R_1 and the image of B is invariant under R_2. Finally, we proved that if R_1 and R_2 have characters χ_1 and χ_2, then the dimension of a homomorphism $R_1 \to R_2$ is equal to $\langle \chi_1, \chi_2 \rangle$. Finally, we discussed conjugacy classes and Maschke's Theorem, which states that if a representation is indecomposable, it is also irreducible.

Non-Euclidean Geometry

Leslie

We talked about two different kinds of non-Euclidean geometry that violate the fifth axiom of Euclid, spherical geometry and hyperbolic geometry.

In spherical geometry, line segments between two arbitrary points are parts of great circles, which are the intersections of the sphere and planes that pass through the center of the sphere. We concluded that the area of a triangle on a unit sphere equals the sum of its angles minus π. Isometries include reflections and rotations, which can be determined by 3 non-collinear points on the sphere, because a point has distinct distances with 3 non-collinear points, and the set of points equidistant from 2 points form a great circle.

For hyperbolic geometry, line segments are either semicircles centered on the x-axis or vertical lines. Isometries include horizontal translations, reflections, and inversions. Using conjugate complex numbers, we see that inversion can take lines to lines. By constructing two semicircles with arbitrary included angles, we can have another line to the limit of a degenerate triangle, thus tiling the hyperbolic plane.

Voting Methods, or Why We Can't Have Nice Things: A Proof

Lucas

In this class, we investigated different methods to try to determine the most equitable way to appoint rulers for the secret space society. On the first day, we were given ballots of 5 candidates in order of popularity. A group of ballots was defined as a ballet. We came up with different methods to determine an overall winner of a ballet. We then decided on guidelines that make a voting method fair. We tried to see which of our voting methods would follow these guidelines, but we came to the conclusion that the only one is dictatorship. This conclusion is similar to Arrow's Theorem, which states that the only voting method with independence of irrelevant alternatives, the Parrot property, a guaranteed winner, and monotonicity is a dictatorship. We formally proved this theorem by first proving a small set of six voting theory lemmas.

On the last day, we created our own space colonies with corresponding populations. Based on the populations of these colonies, we tried to come up with a system to determine the number of representatives that each colony should be allocated. We first tried allocating representatives proportionally, but this proved to be difficult because colonies had to give an integer number of representatives. The U.S. House of Representatives currently has the same problems we encountered with our methods, and they use a method proposed by Thomas Jefferson, which is effective but still not perfect.

Complex Analysis is the Real Deal

Vincent

We started with the well-known identity $e^{i\theta} = \cos\theta + i\sin\theta$, and the well-known proof using Taylor series. However, Taylor series were originally defined for real numbers; how do we know that they still work for complex ones?

It turns out that we can show they do work, as long as the function in question is holomorphic, meaning it has a well-defined complex derivative. A lot of functions we are familiar with are holomorphic, including polynomials, exponentials, sin, and cos (which are just linear combinations of exponential functions).

We then defined a complex version of the definite integral: the contour integral. The contour integral over a path γ is $\int_\gamma f(z)dz = \int_a^b f(\gamma(t))\gamma'(t)dt$.

After that, we spent a few days proving a surprising theorem: if f is holomorphic, the contour integral over a path doesn't depend on the path at all. It only depends on the endpoints.

Using this powerful new theorem, we showed that all holomorphic functions have Taylor series expansions, just like real functions. This leads to Liouville's Theorem, which states that all bounded holomorphic functions are constant. From this, we proved the Fundamental Theorem of Algebra: every non-constant polynomial has a complex root.

The Secret Course

Micaela

The way to encrypt a message in RSA is for the person who intends to receive the message to pick two (public) numbers, n and e, where n is the product of two primes, p and q, and e is relatively prime to both $p-1$ and $q-1$. There is a number d such that $ed = 1 \pmod{(p-1)(q-1)}$, and the receiver must calculate d (this is possible because they know p, q, and e). The receiver (publicly) tells the sender what n and e are, but the numbers p, q, and d are all kept private. The sender then takes the message t (a written message converted into a number), and sends the receiver the encryption, c, where $c = t^e \pmod{n}$. The receiver then calculates $c^d \pmod{n}$, and this number equals t. We answered questions such as: why does $c^d = t \pmod{n}$? How can the receiver calculate d? How do we find large primes? Is it even feasible for the sender to calculate $t \pmod{n}$ on a computer? Which algorithms are more efficient than others for calculating these numbers? The answers to these questions are classified.

Daily Gathers

Superhuman Counting by Sommer Gentry

Maya

Sommer Gentry is a mathematician who works with operations research, trying to make the best decisions possible using mathematical methods. She described herself as a "transplant mathematician" as she is currently looking at redistricting liver-sharing regions. Sommer's Daily Gather was not about organ gerrymandering, however, but about her upcoming appearance on the Fox television show *Superhuman*.

The problem Sommer dealt with on the show involves finding the number of rectangles in an L-shaped figure split into squares. The problem can be made into a simpler question - how do you deal with one rectangular part of the L? - and a harder question - how do you deal with the L itself?

Starting with an $n \times m$ rectangle, you can describe every rectangle by the two vertical lines and two horizontal lines that define its edges. This is represented by $\binom{n+1}{2}\binom{m+1}{2}$. Sommer explained that to count subrectangles in simple rectangles quickly on the show, she memorized products of triangular numbers.

In the end, we considered other problems that involve counting parts of a whole, such as slicing a torus and counting the parts. These problems might not actually use superhuman abilities, but the mathematics involved might be even more fun.

Math Movies

Oliver

Adventures of the Klein Bottle A Klein bottle is a two-dimensional manifold with no outside and inside, which can be created in 3 space with a self-intersection. Like the Möbius strip, the Klein bottle is not orientable. The Klein bottle is created by joining two Möbius strips.

Regular Homotopies in the Plane A *regular curve* is a closed, continuous path in the plane that has a tangent line. A *regular homotopy* is a transformation of a regular curve that does not involve lifting the curve out of the plane, breaking it, creating corners or cusps, or turning the tangent vector discontinuously. The *rotation number* of a curve counts how many net revolutions the tangent vector makes from its starting position when traveling around the curve. Any two curves are regularly homotopic if and only if they have the same rotation number.

Periodic Time Table Optimization In the subways in Berlin, passengers often switch trains, sometimes narrowly missing one and having to wait a while for another. Mathematicians teamed up with a planner in order to minimize the wait time between trains. They managed to eliminate all of the millions of suboptimal schedules, and used computer optimization tools to decrease the average wait time from 2 minutes and 48 seconds, to 2 minutes 30 seconds. Sometimes math can make people happy!

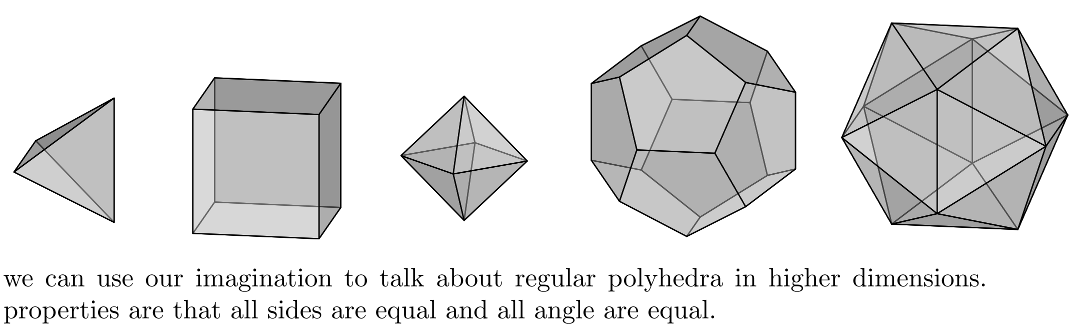
Four-Line Conics In the video, there was a sequence of conics displayed.

Geodesics and Waves A geodesic curve is the shortest path between two points. The geodesic curve on the earth between two points creates paths for planes and ships.

Knot Energies Knot theory focuses on proving two knots are equivalent. One way of doing this is giving an energy for points on the knot. The circle has no energy, and the trefoil has minimum energy. The sum of two trefoils is twice the energy of one trefoil. The idea of energy also works for links.

Regular Polyhedra in Higher Dimensions by Emily Jack, Images by Asa

Can you imagine a hypercube? This daily gather can help you figure it out. First are some platonic solids. What is a platonic solid? It has the following properties: Firstly, all faces are the same. Secondly, it should be a polytope. Thirdly, the angles of its faces are the same (so that all faces are regular polygons). Lastly, it should be convex. Here are all the platonic solids:



Then we can use our imagination to talk about regular polyhedra in higher dimensions. Their properties are that all sides are equal and all angle are equal.

We considered the question of how many regular tetrahedra can you tie around a point and nets of polyhedra. We discovered that there are exactly six regular polytopes in 4D. The properties of a regular polytope are that firstly, all its facets are identical platonic solids. Secondly, it is convex. Thirdly, the number of facets is the same at each vertex. Fourthly, its angles are the same. We asked which regular polytopes have cubes for facets.

A Pause to Reflect by Elizabeth Drellich

Adrian

What happens when we reflect across a line? A number of things. Notably, the line being reflected about maps to itself under the transformation, and that same line is a perpendicular bisector of the segment connecting a point to its image (meaning the normal vector from the line to the point is multiplied by -1 to yield the point's image). Also, if the line of reflection passes through the origin, the reflection is a linear map. An example of this is a reflection over the x-axis:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

If we reflect over the x-axis and the y-axis, the result will not be a reflection, as there is only one point (the origin) that is preserved by the map.

We also found that if you reflect an asymmetrical smiley face over the x-axis and the line $y = x$, the result is a rind/popplewhip/sproGh with eight members, but only four were reflections of the original smiley face. If this is extended to three dimensions, the rind/popplewhip/sproGh forms a fascinating solid akin to the ones we made with Zometools.

Another possibility is to let the normal line be multiplied by a root of unity rather than just -1. We saw how in some cases, this results in an unusual rind/popplewhip/sproGh.

Multiset Poker by Garth Isaak

Zhamilya

Let's say we have a deck of four cards K_1, K_2, Q_1,and Q_2. Pick one, replace it in the deck, and pick a second card. What is the probability of getting each possible hand? What if we have 2 decks of cards to pick from or a multiset, where you can repeat a card in a deck however many times you want? In order to answer these questions we were introduced with new notation. $\binom{n}{k}$ = the number of k element subsets from n set: $\binom{52}{5} = \frac{52!}{5!47!} = 2{,}598{,}560$. $\left(\binom{n}{k}\right)$ = the number of k element multisets from n set: $\left(\binom{52}{5}\right) = \frac{52-5+1!}{52!4!} = 3{,}819{,}816$. $\binom{n}{k_1,k_2,\ldots,k_t}$ = multinomial coefficient: $\frac{13!}{1!2!10!} = 858$.

By answering these questions, we found that the only fair game was a multiset. Therefore, our goal was to learn how to play a multiset poker with a 56-card deck. What we did is we tried to find a bijection between playing poker with a 56-card deck where the four extra cards are called knights and playing a multiset poker with a 56 card deck. Picking 5 different non-knights in a multiset corresponds to picking 5 different non-knights in a 56-card poker. You can order the 5 cards you pick in a certain way, and if you have one or more knights in a row, they will take the identity of the card that is in the spot to the right of them. Thus, there will be a 1-1 correspondence between a 56 card poker and a multiset poker.