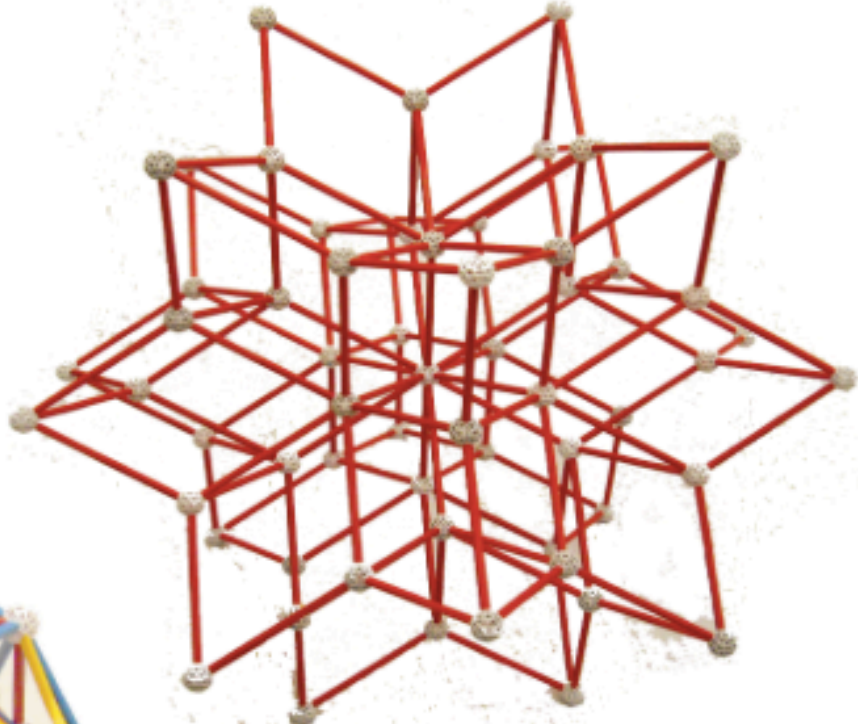
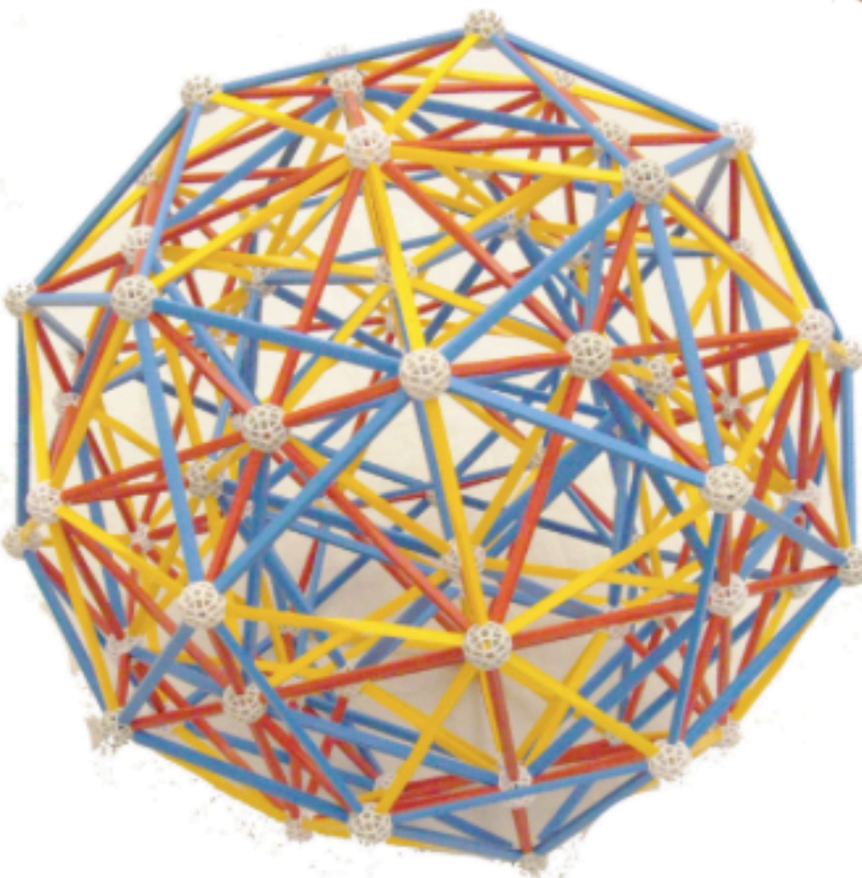
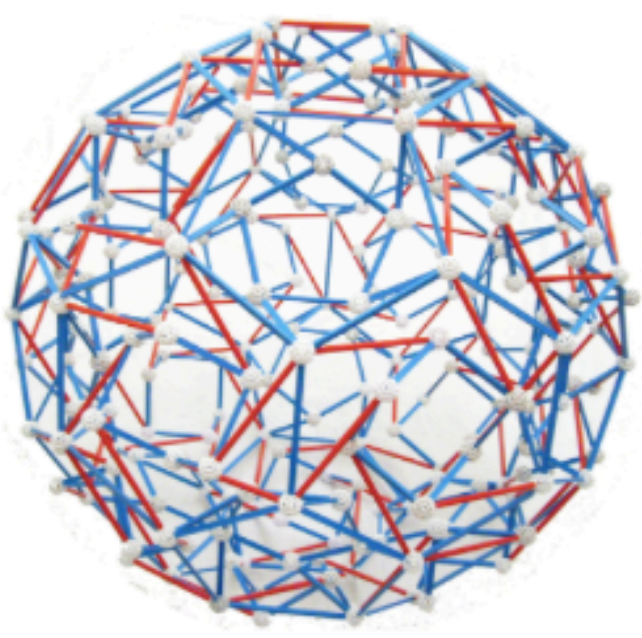
# MathILy

## Record of Mathematics

Issue 2







## Tuesday Daily Gather: Fibonacci. by Brian

Jonah

Brian's brother has a favorite positive integer $n$, and he wants to find a Fibonacci number $F_k$ such that $n \mid F_k$. Obviously, the trivial case $F_0 = 0 = n \cdot 0$ satisfies this condition, but Brian's brother wants actual results from actual math. How do we prove that such a number exists for any $n$ Brian's brother can choose?

Given two pairs of adjacent Fibonacci numbers $(F_{k-1}, F_k)$ and $(F_{j-1}, F_j)$ for which $j \neq k, F_{k-1} \equiv F_{j-1} (\text{mod } n)$, and $F_k \equiv F_j (\text{mod } n)$, we know that the Fibonacci numbers mod n will begin to repeat. These two pairs can always be found because the number of possible pairs in $\mathbb{Z}_n * \mathbb{Z}_n$ is finite, while there are infinitely many pairs of consecutive numbers in the Fibonacci sequence. We can also extrapolate the cycle backwards, so the entire Fibonacci sequence cycles in mod n; the cycle has no beginning other than the beginning of the sequence. This means that the trivial case of $F_0 = 0$ is also repeated at some point in the sequence mod n. If $F_l \equiv 0 (\text{mod } n)$, $n|F_l$ and we have a solution. Thus, $\forall n \in \mathbb{N}$, $\exists$ a Fibonacci number $F_l$ such that $n \mid F_l$.

**Questions:**

- What if the sequence doesn't start with $(0, 1)$?
- How long is the cycle for some $n$?
- Can I always find a number $F_k$ such that $F_k \equiv r (\text{mod } n)$ if $r \neq 0$?
- If n is prime, will all possible $(a, b) \in \mathbb{Z}_n * \mathbb{Z}_n$ appear in the cycle?

Claim: If $n = p_1^{a_1} \cdot p_2^{a_2} \cdot ... \cdot p_k^{a_k}$ where all p are prime, $L_n$ (the length of the Fibonacci cycle mod n) is LCM$(L_{p_1^{a_1}}, L_{p_2^{a_2}}, ..., L_{p_k^{a_k}})$. If $n = p^k$, $L_n = L_p \cdot p^{k-1}$.

Any composite $n$ must be either the factor of two coprimes $a$ and $b$ or a power of a single prime, because any two sets of primes in its factorization are coprime. First, assume the former: $n = a \cdot b$, $LCM(a, b) = 0$. If $L = LCM(L_a, L_b)$, $F_k \equiv F_{k+L} (\text{mod } a)$ and $(\text{mod } b)$, so we can see that $L$ is a multiple of $L_{a \cdot b}$ and, since $gcd(a, b) = 1$, a factor of $L_{a \cdot b}$. Being both a factor and a multiple, it must be equal, so we know that $L_{a \cdot b} = L = LCM(L_a, L_b)$.

If n is a perfect power (neither a prime nor a multiple of two coprimes), $n = p^k$, $L = L_{p^k}$ (p is prime). $n = p^{k+1}$ may have the same L as $n = p^k$, but if it does not, $L_{p^{k+1}} = p \cdot L_{p^k}$. After this happens once (that $L_{p^k} \neq L_{p^{k+1}}$), it will happen for all subsequent powers of p. Brian determined this through a series of recursive steps which he has described as long and not particularly interesting; they have been omitted here for the sake of brevity.

## Wednesday Daily Gather: Math Movies

Lavinia

The first movie we watched is called "linear programming". This movie is about a man that wants to steal bricks to get the max profit. There are two types of bricks. The first type is small but heavy, and the profit for each brick is of $5. The second brick is big but light, and the profit for each brick is of $6. But there's a problem: the cart that the man is using has a max volume and a max weight that it can bring. The man tries to modify the number of bricks but fails miserably, so the horse decides to help him. Using a graph to find the intersections of the lines that map the max weight and max volume of goods he could bring, he finds out how many bricks of each kind the man should take in order to get the max profit. The second movie we watched was about isometries: An isometry is a distance-preserving transformation. Reflection, rotation, translation and glide reflection are isometries. Then we watched a video about dancing squares, triangles and rectangles. Then one about homotopy and embedding. And one about soap bubbles. This last one explained how eversion (turning something inside out) works in spheres. Finally, we watched an amazing video about the evolution of virtual block creatures. Some of the tasks that these digital creature could do was compete for green blocks, swim, and fly.

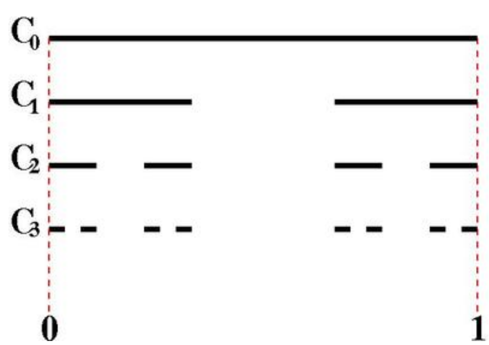
## Thursday Daily Gather: Journey into Induction by Robert Vallin

Wen

In this Daily Gather, by Dr. Robert Vallin of Lamar University, we were taken on a journey through various problems he had encountered in his mathematical career. The first problem regarded the Cantor Set. The Cantor Set is created through a recursive process, starting which a solid segment of length 1. You then take the interval, and remove the middle 1/3. So, you get something that looks like this:



Then, you repeat the process ad infinitum:




The next problem he posed was the following: Given a 9x9 chessboard, remove 2 squares from opposite corners, as well as the central square. Is it possible to cover the remaining squares using 2x1 dominoes? This problem was one he gave in the Dead Poet's Society at Slippery Rock University.

In the next segment of his daily gather, Dr. Vallin transitioned into a magic trick, called the Red/Black Trick. He had attended a math magic event, and enjoyed the union of math and magic. He allowed the audience to cut the deck. He then asked for a number of cards

to count off the top. He then riffle shuffled the cards, put them behind his back, and pulled out cards two at a time: one red, one black. This "magic" trick has mathematical workings behind it. For the trick to work, the deck needs to be pre-arranged, with alternating red and black cards, and alternating suits. Cutting the deck is an illusion - wherever the cut is, it will be put back on the deck and preserve the red/black order. Additionally, by counting off the cards, it reverses the order of the top cut, a phenomenon which does not normally occur with a normal shuffle. Through these means, all he had to do was take the top two cards of the deck to get one red, one black. This shuffle, the Gilbreath shuffle, is named after the mathematician Norman Gilbreath. Mathematically, it can be described with a Gilbreath permutation, which is a permutation of {1, 2, 3, ..., n} in which the first j elements are consecutive integers. We were then presented with two problems.
Problem 1: Show that there are $2^n$ Gilbreath permutations of the set {1, 2, 3, ..., n}. One solution was found in which a bijection between consecutive numbers and a sequence of up and down arrows corresponded to an increase or decrease between consecutive elements.

A second problem, which is a lemma for a proof related to Gilbreath permutations, was also posed: Show that $2^n = n + 1 + \sum_{n=1}^{n} 2^{k-2}(n - k + 1)$.

While no solutions were posed during the daily gather, numerous proofs were given before the start of life seminar, two days later.

## Friday Daily Gather: Let's Take a Walk by Corrine

Khunpob

In this Daily Gather, we looked at expected value of reaching a specific point during a random walk, whether it be in a straight line or in irregular shapes. The first example was if Baby Dietmar was lost in the woods on a number line, and there was a cliff at -100 and his house at 100, what is the expected number of steps before he reaches either end of the number line, given that he will either step left or right with equal probability. It turns out that the expected number of steps is 10,000. This can be calculated by considering how expected number is linear. Therefore, the expected number of steps at any given position can be expressed as the expected number of steps from the two spots to the left and right of it. By doing this, one can continuously write the expected number of steps at any given position until one reaches the end, where the value is known.

Then, variations of the question were considered. One of them was, instead of the cliff and house being equidistant from Baby Dietmar, if the cliff was 50 steps to the left and the house was 200 steps to the right. While not proven, the formula was generalized such that if Baby Dietmar had to take $S$ steps to the left to get to the cliff, and there were $N$ steps between the cliff and the house, then the expected value is $S(N - S)$. Other examples of nonlinear maps were considered. One of them was it was a cycle. What is the expected number of steps needed to get back to the original spot Baby Dietmar was on? In this case, one can cut the cycle at any point, and then expand it into a straight line to be answered using normal methods. Lastly, we considered the example of a pentagon with all the diagonals drawn. Two points A and B were labeled, and we considered the expected number of lines passed before A gets to B. In this case, as any two pair of points are connected, the expected number of steps from any point to B is the same, which allows us to solve for it in a system of equations.