mathily-er

WEEK OF CHAOS

ROM issue 3

editors: Casey, Lindsey, Wren

Introduction

This special edition of the ROM attempts to document the disorderly disarray of the Week of Chaos. In it, you'll find class and Daily Gather summaries by various students, a short story by Cindy, two puzzles by Jonah, and of course, quotes of the week.

The font is Garamond, and as this chaotic week has so much content, we've set it to size 11.

Enjoy!

Week of Chaos Classes

Algebraists Anonymous: The Chad Support Group for the SuffeRING with Alice

By Boris

The first step to recovery is admitting you have a problem. In the first meeting of our support group, we discussed the problem that plagued us all. It goes by many names: some call it a swirly boi, others a turkey, and sometimes a toi. Regardless, the symptoms are the same. Those suffering from toiz syndrome are quick to associate with others; have an over-uniform distribution of body fluids; sympathise with communists; have multiple identities, one of whom is Samantha; and believe certain doom constantly looms over them.

Our goal for these sessions was to learn more about our problem and to recover by becoming a Chad. A Chad does not suffer from toiz syndrome. People cool enough to be Chads are cool enough to associate with just enough people, have only one identity, and can do things in reverse. During this week, we've diagnosed different sets with toiz syndrome and Chad, permuted our understanding of sigmutations, created functions that mapped Chads to each other whilst preserving their behaviour, modded the Basie Boiz, meet new families of subChads, and even did a bit of Chad sudoku. We've spoken to old friends, like the integers and S(oz)oms, and made some new ones along the way, like the sigmutations and torchifiers.

Although we've developed a deeper understanding of toiz and Chads, we still don't know everything. There remain questions unanswered, conjectures unproven, and jello swirls untouched. Regardless, we've learnt many things, and together, we can recover from toiz syndrome and become Chads.

Alternating Sign Matrix Conjecture with Jonah

By Cole

The class began with the introduction of alternating sign matrices (ASMs): matrices with zeros, ones, and negative ones, following two rules: the numbers in each row and column sum to one, and the nonzero numbers in each row and column alternate between 1 and -1. Through calculation and the knowledge that all Szams/Samanthizers are ASMs, the number of 1x1 through 5x5 ASMs were calculated, which were 1, 2, 7, 42, and 429 respectively. These calculations showed that the number of ASMs grow surprisingly quickly, there being 7436 and 218314 ASMs for 6x6 and 7x7 respectively.

After this, two new concepts were introduced: pipetown and square ice. In pipetown, there is an n x n grid of tiles, each tile with a pipe that either goes straight or curves 90 degrees. All pipes must be oriented so they do not end suddenly, and exit pipes must alternate on the border. In square ice, there's an arrangement of n x n oxygen atoms, hydrogen atoms between every pair of adjacent oxygens, and additional hydrogens on the beginning and end of each row. Molecules of water must be drawn so that bond angles are either 90 or 180 degrees. Problems in math are always related, so the Bijection Junction between pipetown, square ice, and ASMs began.

After Bijection Junction, Jonah the introduced a totally (un)related problem: linear breakings. The linear breakings of n are the ways to split n into the sum of smaller numbers such that each number in a summation is greater than or equal to the number it precedes. Interestingly, the number of n-breakings with 3 or fewer numbers equals the number of n-breakings with each number being less than or equal to 3, which was proven using a bijection involving bar graphs. The ASM conjecture had yet to show its face, but there was a new problem on the horizon: plane breakings.

Plane breakings are like linear breakings, but instead of a standard sum, the numbers are arranged in a grid and the sum of all numbers in the grid are n. These plane breakings could be represented by a 3D bar graph. This is where the final question showed up: how many plane breakings are there such that biggest number, the number of rows, and the number of columns are less than a certain value n; and when plotted on a hexagonal grid has perfect symmetry (60-degree rotational symmetry and 6 lines of reflective symmetry). The number of these plane breakings for a value 2 is the same as the number of n x n ASMs. Even more remarkable, the reason for this relationship has eluded mathematicians to this date, despite this relationship being found before ASM conjecture had even been proven.

Axiom, I Choose You! with Jonah

By Zachary

There are 100 mathematicians in 100 identical rooms. In each room, there are infinite boxes, labeled 1 through ∞. In each box, there is a random real number. Each mathematician opens all boxes except one of the boxes, then guesses the number in the box they left closed. 99 out of the 100 mathematicians will guess the number in their box without fail. How can this be? This is was the question Jonah posed to us on the first day of class. However, in order to answer this question, we had to start on a smaller scale. We considered what would happen if instead of random real numbers in the boxes, there were finite black and infinite white marbles. In this scenario, the mathematicians all had different rooms, but could see the marbles in the other rooms. Since there were a finite number of black marbles, the mathematicians would look into the other rooms and see at what point there was the very last black marble. Then they would guess that the box after that box had a white marble. Using this strategy, we were able to generalize to more complex variations of the problem until we finally made it up to the original problem Jonah wrote on the board. After completing the real numbers in boxes problem, we did the Banach-Tarski paradox problem as well as an infinite version of the hat color game. You may be wondering, “how do 99 out of the 100 mathematicians always guess the correct box?” Try and find a strategy yourself.

Cardinal and Ordinal Numbers with JD

By Brandon

In Cardinal and Ordinal Numbers, JD introduced us to the strange world of infinite sets, and how some infinities are more infinite than others. We began by examining sets; what one was, what is required for a bijection from one to another, and what it meant for sets to be equal in size. In addition, we noted the definitions of addition, multiplication, and exponents for sets and proved they were good definitions, and explored power sets. We also noted how the composition of bijections is a bijection, and saw that cardinal equality was transitive and symmetric. Beginning on Thursday, we explored the many and varied properties of finite and infinite sets of natural numbers, and looked at the cardinality of the set of natural numbers, and creating a bijection between it and the integers, then moved on to examining the rational numbers and finding a bijection between $\mathbb{N}$ and $\mathbb{Q}$. Friday, we finished off the course by proving that the set of real numbers is uncountably infinite; it was larger than the set of natural numbers. In addition, we proved that the set of reals to the power of the set of reals, CC, was strictly larger than the set of real numbers.

Forbidden Colors with Connor

By Saraphina

When we first walked in the door on Monday, we were met by a pitch-black room with Connor sitting calmly at the back. This ominous environment was very àpropos for the ominous environment that was our forbidden colorings. We were introduced to a game where two players take turns filling in an edge on a 6-titanic with two colors, and the first one to complete a monochromatic 3-titanic (triangle) loses. We then proved that a winner is guaranteed for this game. We call this concept "star numbers," formalized this as "the minimum number of vertices *m* such that a red *r*-titanic or a blue *s*-titanic is guaranteed on any 2-coloring of an *m*-titanic" and notated this as $☆_{r,s}$ = n. In this case $☆_{3,3}$ = 6. From there, we found that the star numbers $☆_{3,4}$ = 9 and $☆_{4,4}$ = 18, and proved them. We continued this for the next couple of days, finding lower bounds with probability instead of bashing out examples. On Thursday, we were introduced to a new problem: finding the minimum length of a list such that the numbers create an arithmetic progression of *k* terms. The class struggled to find a way to connect this problem to star numbers and generalize reasoning from specific cases to general arguments. The next day, we returned to star numbers, found a general upper bound $☆_{r,s} \leq ☆_{r,s-1} + ☆_{r-1,s}$ and proved it together as a class. As it turns out, an exact formula for these numbers doesn't exist. In fact, star numbers are one of the hardest problems out there. All that's known about $☆_{5,5}$ is that it's between 43 and 48, and the numbers after that have even broader bounds. As for the arithmetic progressions, it turns out they really don't have a direct connection to the star numbers, nor a pattern or even many more known terms. Both of these problems are in a branch of combinatorics studying the minimum number for certain patterns to exist, and these numbers are infamously difficult to discover. Naturally, we didn't find many values, but we did make substantial progress in understanding and discovering bounds for these problems.

Generating Functions with JD

By Alex

Generating functions this week was pretty awe-sum! We started on the first day with a closed form of the Fibonacci sequence. We got more comfortable with summation notation and rearranging/combining summations. We got an introduction to the essence of generating functions when we learned that the sum as k goes to infinity of x^k is equal to 1/(1-x). We then used this to find the sum as k goes to infinity for many variations of x^k. Since we didn't have to worry about convergence, we could freely calculate infinite summations. Throughout the week, we continued and elaborated on the idea of how to add generating functions, as well as finding neat closed form expressions for other sequences. For instance, after laboring on Thursday and Friday for more than an hour, we finally discovered the closed form expression for the Catalan numbers, and in the process we disproved a math paper that JD was using for reference. We also found that a neat generating function came out to 2^n + 3^n. Another major part of our class was exploring partitions (that is, how you can break up integers into sums of smaller integers). We broke it up into smaller cases at first, such as a max size for partitions; finally, after lots of toil and neat rearrangements, we found the expression for the number of integer partitions for an integer n based on its generating function. This class was great because we used astute algebra skills to manipulate generating functions to get astounding results; and we were able to do lots of cool math without using calculus which is usually an *integral* part of generating functions.

Hard Problems with Connor

By Julia

We began with a simple goal: by the end of the week, we will have proven *P*=*PV*, that the problems that are polynomially solvable (*P*) are polynomially verifiable (*PV*). Easy enough.

The Scales and Rocks: You have an arbitrary number of rocks (which are indistinguishable, and vary by weight) and a scale. You must sort the rocks in ascending order by weight. The class came up with some neat sorts to solve the problem: **The Irma Sort**: Compare the first rock to the rock to the right of it. Switch if the left rock is heavier. Continue until the heaviest is at the end. Then, repeat. **The Partition Sort**: Take one rock and compare it with all of the other rocks, separating into piles that are heavier than that rock and piles that are lighter than that rock. Then, repeat this step, but choose a rock in one of the sub-piles and create sub-sub-piles using that one rock. Repeat until complete. **The Merge Sort**: Split the rocks up into easily sortable chunks. Then, put two easily sortable chunks together by taking the first element of each chunk and comparing them, putting down the lighter one. Repeat this step until those two chunks are sorted. Continue putting chunks together until it is sorted.

The Ambassador Problem: Let's say we have a dystopia of *m* cities and *n* ambassadors. We want to place those ambassadors in the dystopia, but we cannot place two embassies in one city or adjacent cities. We were told to find an algorithm to determine where to place ambassadors, and the only solution we could find would take *m* choose *n* steps to solve the problem.

The Coloring Problem: If given a dystopia *G* of *n* cities, can we figure out whether the dystopia's cities can be colored red and blue, with none of the adjacent cities having the same color? We solved this by isolating every city by coloring it red, and the adjacent cities blue, and then seeing if that coloring is valid. This algorithm took about *n* steps.

P: We then reconvened and discussed. We learned that *P* stands for polynomial time, which means that a problem is solvable in x^n steps. Linear and quadratic time fall under polynomial time.
The Magic Box: Let's say we have a magic box that solves the ambassador problem. If given a dystopia *G* and *n* ambassadors, it returns a valid ambassador configuration or the word no. Can we easily verify that the configuration is valid? Yes! Simply check every embassy and make sure there are no adjacent embassies.
PV: *PV* describes a problem that can be easily verified in polynomial time, but does not have a polynomial time solution. Just like we could easily verify a solution to the ambassador problem, but we couldn't find a quick way to generate a valid ambassador configuration. We also argued that *P* is a subset of *PV*.
Watchtower Problem: Given a dystopia *G* and *n* soldiers, place the soldiers in watchtowers in such a way that all bike paths are seen. You may see all bike paths adjacent to the village. Watchtowers may be adjacent as well. We decided that this problem was the same as the ambassador problem, but with the villages switched around (place an ambassador where a watchtower isn't). We decided that, given a magic box that solves the ambassador problem, we could also solve the watchtower problem, and vice versa.
3SAT Problem: Given a boolean expression, we were told to find a configuration of booleans that make the entire expression true, where 1 means true and 0 means false. Say we have the expression (*a* or *b* or *c*) and (not *a* or *b* or *c*), where you take "or" statements called clauses with three variables and "and" them all. This problem is also a problem in *PV*.
NP-Completeness: 3SAT is one of the hardest problems in *PV*, meaning that it is *PV*-Complete, so if one could find a polynomial time algorithm for it, one would theoretically have a polynomial time algorithm for every problem in *PV*. We went on to prove that the 3-coloring problem was also *PV*-complete.
The class adjourned, *P* vs *PV* still unsolved.

Hyperplane Arrangements with Jonah

By David

During the Week of Chaos, we had much fun in HYPERPLANE ARRANGEMENTS with Jonah! We started the class by a pretty intuitive problem: how many regions do the x and y-axis of the coordinate system divide the space into? Then what about in 3 dimensions? 4D?... 7D? ... nD? The problem seems pretty hard to visualize when it comes above 3-dimensions. However, after we developed how to notate axes and define regions in space, the problem becomes easily solvable algebraically. Afterward, Jonah introduced more challenging problems. More combinations of hyperplanes appear in the problems which make them more and more complicated. We successively explored "Normal Arrangement," "Diagonal Arrangement," "Pickle Arrangement," "Buffet Arrangement," and also "Unknown Arrangement." Not only did we visualize hyperplanes when counting, but we also found many interesting connections from hyperplane arrangements to combinatorics. "Pickle Arrangement" appears to have some truly magical relations with Catalan (or Cat and Lamb) numbers. Jonah also introduced to us two "unrelated" problems: "Picky Sams at a Buffet" and "Picky Sams at a Circular Buffet," which has to do with the counting in "Buffet Arrangements." As for "Unknown Arrangements," its explicit formula of counting is still unknown in mathematics just as its name states. In the last class, Jonah introduced two new problems in combinatorics--"Long Chess Board" and "Branded Broccoli Bifurcation." After exploring different cases in those two problems, we surprisingly found out that every case maps to a counting result of a hyperplane arrangement. The relation between the problems and hyperplanes, according to Jonah, is still a mystery. After five classes in the week of chaos, we ended up having so much fun and thoughts about the exciting hyperplanes!

Markov Chains with JD

By Davis

"Since D = B, D = A you dinglenut." - Rishav

To start off our second week of chaos class, JD went around the room asking each of the nine students to give him a sequence of three coin flips before countering it with one of his own. Then, we flipped a coin a ton of times and whoever's sequence appeared first, JD's or the student's, won. Out of the eight cases of what a student said, JD's counter-sequence won first six times. We tried to calculate individual case probabilities, but we had to wait until we learned about Markov Chains to really understand it.

Our next problem told us that there are three types of weather in Seattle, each defined by the weather the day before: sunny, rainy, and very rainy. We were first asked what the probability of it being sunny on day four is if day one was rainy. And then on day 100. To do this we tried to define patterns and formulas, but they fell apart after more than ten days. Our final solution actually used matrices raised to certain powers which also allowed us to change the probabilities of each type of weather based on the day before. Now that we knew that our solutions required us to use BOOMZ (between 0 and one matrices), we were introduced to another problem.

JD House is at the House house and wants to go to a party. Each unit of time, he moves one road forward or one road back, each equally likely. There are three roads between his house and the party. If he gets to either his house or the party, he stops and spends the night there. What is the probability that he makes it to the party? Since we were able to use logic and counting to figure out this specific case of three roads, JD changed the number of roads between the House house and the party, so we had to use BOOMZ. As we defined these matrices and manipulated them, we were able to create a summation and eventually a multiplication of three matrices that we could put into a calculator. Thus, we could then solve this problem for any number of roads.

Our week of hard work ended so perfectly as we looped back to the coin problem on the last day. Now, we could be more accurate in our definitions and use BOOMZ to figure out the exact probabilities of victory for sequences of different lengths or even see how a weighted coin would affect these probabilities. At the end of the week, we understood why JD chose the sequences he did, what Markov Chains are, and the solutions to our three problems.

Methods of Proof with Cathy

By Cindy

Everyone in this class basically signed up for a short version of the evening class where you get a problem set and write proofs.

As usual, we got two anagrams up on the top of the page. The anagram "Such a Myth!" can be rearranged into "Hasty Much?" or "Cushy Math." Likewise, "Fed Smooth Prof" yields "Methods of Prof" or "Pfft! ROM? Shooed!," which sounds somewhat derogatory.

Everyday we learned a new way to prove things and then spent the rest of class working on proofs. The only exception was Friday, when we spent all class working on proofs. We covered induction, bijection, contradiction, if and only if (IFF), the following are equivalent (TFAE), and the contrapositive.

Bijection involved proving that two things both counted the same thing while induction was more about showing that if something always worked for the next case, it worked for all of them. TFAE just meant assuming one statement and using it to prove the next one, assume that one and prove the next one, and so on until you get a circle of statements equivalent to each other. The contrapositive was swapping the if and then clauses of a statement and negating them. It's a useful tool for proofs in general.

While we were working, Cathy would walk around and check up on everyone and read proofs. We also got Sam the entire week and JD came pretty often to help.

Alice says her favorite method was contradiction because she's quite contrary and her garden grows with silver bells and cockleshells. Contradiction is fun because you assume the opposite of your claim and take it to its ultimate conclusion where the universe finally shatters into many tiny pieces.

Number Theory with Cathy

By Harry

Exploring Number Theory first started with exploring Communism. Specifically, Tois that were Communist. These Tois also had several other properties that we had defined before in Root Class. After defining Tois and Basie Bois, we were on a roll. Thus, we proceeded to define and rename every term about Tois we had previously named in Root Class. Szalmon, Celfies and Stoqqer (Pronounced stop-er). After this, we were finally able to tackle our big question. What are all solutions (X,Y) to the equation $X^2 + Y^2 = P$ Where P is a prime number. We first found that for there to be any solutions, P must be able to be written as 4n+1 or be equal to 2, which Kenyon was able to prove. After this proof, we made a couple more conjectures before Ben made a joke. He factored $X^2 + Y^2$ as (X+iY)(X-iY). Little did he know that his joke was in fact not a joke, but a true fact in the Joke. We first proved that the Joke is a Communist Toi. We then explored jokes further. We talked about the size of the elements of the Joke, finding that size was preserved though multiplication. Using this, we were able to prove that the PLOOMS were only 1, -1, i and -i. We were then able to further define Ploomy-bois within Jokes. We found that Ploomy-bois could not be written as a product of non-Plooms. After proving this, we were able to solve the big question. For a prime to be able to be written as X^2+Y^2, it must not be a Ploomy boi. However, we still needed to figure out how to find Ploomy-bois, so we decided to try visualize these Jokes. We called these visualizations MEMES. We noticed some patterns in memes. When we graphed the lines for mod n in the complex plane, where n was an element of the Joke, we would get an infinite number of squares that extend across the entire complex plane. We named this the web. If we had two relatively prime elements of the Joke, we were able to get the entire Joke. After exploring division in Jokes, we decided to explore division in different sets, specifically $\{x+ \sqrt{5}\,yi: x,y \in \mathbb{Z}\}$. Confusion ensued. First of all, we found that if x^2 has a factor, x does not necessarily have that factor. That’s like saying that 81 is divisible by 3 but 9 is not. Not only this, prime factorization was not even unique in this set. One number could have multiple prime factorizations each with different primes in them. We then looked into the applications of what we learned. We found that memes are extremely useful in real life. In fact, they are used to create elliptic curves for elliptic curve cryptography. These curves are many times more efficient than RSA, making them the possible future of cryptography. Who knew looking at memes all day could actually help the progress of humanity.

p-adics with Cathy

By Wren

To quote an anonymous student in the *p*-adics class, “What *are p*-adics??” Yet, this quote was taken from merely the first day, where the class explored how to construct real numbers from the rationals. A strategy was soon created by taking an infinite or finite list of integers such that when we multiply the first term by 10^0, the next by 10^{-1}, etc. and sum them, we get a real number. As this was suggested by Brandon and Jack, whose ship name is “Brack,” the strategy was named the “Infinity Brack.” As “Brack” sounds slightly similar to the 44th President of the United States, Barack Obama, the real number generated by the “Infinity Brack” was named the “Obama.” Following along this theme, the class defined an “Infinity Biden” as a list of rational numbers such that after a certain point in our list, the values, also known as “constituents,” get as close to each other as we want. After two hard days of WOC, another anonymous student in the *p*-adics class asked a familiar question, “What *are p*-adics???” On Day 3, the question was answered, at least partially, when the class defined a *p*-adic distance by the formula $d(x, y) = |x-y|_p$, where $|x-y|_p = (\frac{1}{p})^{pres(x-y)}$ and $pres(x-y) = n$, where $n = p^n(\frac{a}{b})$. I know, I know, this makes no sense. What makes even less sense is what it actually means: that when p=3, 3^{1000} is a tiny number and 3^{-1000} is a massive number. Once we had attempted to understand this, we generated examples of *p*-Infinity Bidens and *p*-Infinity Bracks. After hours of confusion and at least a few of tears, we arrived at a big, exciting, and even more confusing discovery: that i is a 5-adic number and $\sqrt{2}$ is a 7-adic number. So, this begs the question, “What *are p*-adics????”

Reflection Groups with Alice

By Jack

Reflection Groups was started by examining the reflections and rotations of equilateral triangles. A table was made of combinations of transformations, and the result was something similar to a swirly-boi/turkey, but with only one operation. After name suggestions such as chicken and swirly-lad, the chad was born. The class then worked on finding chad starter packs, consisting of pocket element transformations which can be used to find the whole chad. If a chad’s pocket elements were all reflections, it was a reflection chad. Later on, the class did the opposite, finding the whole chad when given just the starter pack reflections. The class also worked on finding symmetries and starter packs of not only 2D but also 3D shapes, including tetrahedrons and Rubiks cubes drawn on with stolen dry erase markers. Pizza slices, the space between lines of reflection were also studied, and the class tried to find how to create all pizza slices of a shape when starting with only one. An overall question the class worked towards answering was: How many reflection chads are there and what are they? It quickly became apparent that there were way too many possibilities, so the question switched to a more manageable one: How many *finite* reflection chads are there and what are they? Although there were still infinite possibilities in 2 dimensions, there are actually only finite possibilities in higher dimensions.

Surfaces with Connor

By Will (?)

Editor's note. This text is transcribed from a notebook found on the floor of Adams 208, with the title "Official Surfaces ROM article (Will)." No student named Will is present at MathILy-Er this year.

Day 1. Oh God, I think I'm lost. I've been hiking on the Möbius Trail in Surfaceland for hours, but it feels like I'm just going in circles, and this place is freaking me out. I swear to God I saw a coffee cup lying next to the trail, but when I looked closer it was actually a donut, but when I looked closer again it was actually a cup! And when I picked it up, I found it was strangely squishy and stretchy, but not puncturable, pinchable, or ptearable at all. Then I walked into a bunch of polygons just floating around, folding and twisting and, ugh, identifying their edges together, and I saw things that I can't unsee. I saw a square turn into a sphere, and I saw a horrible cruller cake that was actually a tube that went into itself - but when I stuck my finger inside the tube (I was curious), my finger was actually on the outside! Do the plants here give off hallucinogens?

Day 2. There are more of those folding shape things around now, and they just keep getting weirder. It shouldn't be possible to have a two-sided polygon, but they're right here in front of me. It seems that based on their edge identification, some turn into spheres and others twist themselves into some Godforsaken higher dimension. There's other ones too, just zipping and unzipping themselves when edges are identified next to each other. I tried to make a two-hole donut with some macaroni, but it all just melted away through my hands. I must have gone mad long ago, and now I've gone so mad that I'm in a whole new dimension of sanity.

Day 3. I found out that I can cut these floaty shapes up into multiple polygons, with a new pair of identified edges, and they're fine as long as I glue them back together later. That lets me put together a donut with two holes, or probably any number of holes, but then I realized I don't have a formal definition of a hole. So that's pretty disconcerting. I have a feeling I'm going to be stuck here for a while, but at least I won't go hungry.

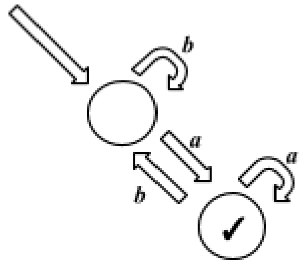
Day 4. May God save us all, because I just saw these crazy surface donut freaks adding themselves together, and they're COMMUNISTS! They can add together by folding one up and then placing the other one onto it, or by cutting tiny holes in the middle of both and joining them by a tube. And the scariest part is these are exactly the same thing. If I can't get out of this place, maybe I can at least stop them from getting out too. But to do that I'll have to first figure out holes, which I haven't made any progress on. So I got away from the shapes and started looking for someone who can help, and I've found a bunch of large, dystopian-looking castles which seem promising. I've been breaking down walls to get into each room, but all I can find is valuable artifacts and no people who might know more about the shape things. And these castles don't feel quite right either - some are built on top of donuts, and some of the walls are actually cuts. It turns out the relationship between the number of towers, walls, and rooms of each castle varies somehow depending on what it's built on, which I do not like one bit. I'll just have to keep looking.

Day 5. I have very bad news. First, I found an ancient stone tablet with a table inscribed on it, and the table seemed to contain all possible surfaces expressed as donuts and crullers added together, although I haven't proven it. Some of them looked pretty scary, and it doesn't seem like they can be stopped. So there's no hope for the rest of humanity. And second, there's no hope for me because I just remembered that Surfaceland is an intrinsically 2D space that doesn't really exist within some larger 3D space. So I'm trapped here, just walking around eating fried pastries forever.

A Tour of Turing with Jonah and Connor

By Taryn

"That's what I wrote, so that's what we're calling it," Jonah informed us as he underlined the words, 'Cool Machines,' on the chalkboard. We soon learned that Cool Machines are series of functions that "read" words and tell whether they are **good** or **bad** (don't say bad words). Each machine is built for a certain alphabet, denoted Σ. A Cool Machine for Σ $\{a, b\}$ that reads only words ending in 'a' as **good** would look like this:



After working with Σ $\{a, b\}$ for a short while, we progressed to more complicated versions, like Σ $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. We built machines for which **good** 'words' were strings of digits such that the number was divisible by 1, 2, 3, ... , 7. Advancements were made throughout the week, such as simplifying machines into tables (a process not widely accepted as 'simplifying' and never fully defined) and composing machines with the combined traits of the parent machines. A splitty machine was defined as a cool machine with a 'psychic arrow' such that the arrow can 'choose' which way to go.

When again confronted with the pickle/lamb problem of root class, we quickly found it to be much more difficult to accomplish using only Cool Machines. An upgrade was needed. Therefore, our friendship with Cool Machines was officially over, and a new friendship began with Really Cool Machines. Really Cool Machines were able to track multiple possibilities simultaneously by allowing the same letter from the alphabet to point in multiple directions. Unfortunately, Really Cool Machines (RCMs) were exposed as the Cool Machines in disguise that they are after they made no headway in solving the pickle/lamb problem. Fake friends.

We needed something more. Something to find pickles, lambs, palindromes, and even repeating sequences. We needed memory. Thus, the Cool Memory Machines were born. Cool Memory Machines allowed new functions that stored a letter in the memory 'fridge' for later consultation. Using Cool Memory Machines, Really Cool Memory Machines, Extra Cool Memory Machines, and even Double Cool Memory Machines, we were able to solve each problem. As the last class of the last day, *A Tour of Turing,* ended very nicely with the conclusion of a machine known as the Turing Machine, the namesake of the class.

Daily Gathers

Monday's Daily Gather with Jonah and Alice

By Rishav

Last week, Jonah and Alice proved they were telepathic. When we found a mathematical method which happened to give the same results, we started to doubt them - could they actually read each other's minds? To prove to us that they were actually telepathic, Jonah pulled 8 forks out of his back pocket and showed them to the entire class. After Alice left the classroom, he let one student flip each fork to either point up or down, randomly, called another student to flip one fork to the opposite direction, and then flipped one fork of his own. When Alice came back into the room, Jonah transmitted which fork the second had flipped to Alice and she guessed it correctly! What was this black magic? We had no idea, so Jonah suggested we try with fewer forks - first 2, then 4, and then 8. With 2 forks, we were able to figure out a method to easily replicate Jonah's and Alice's telepathic communication: Jonah could ensure the left fork was pointed up. Then if the student flipped the left fork down, Alice would be able to guess it correctly. However, if the student chose to flip the other fork, Alice would see that the left fork was still pointed up and guess that the student flipped the right fork. However, the method didn't exactly work for larger numbers of forks. After a lot of thinking, Jonah gave us a hint - just like the method for 2 forks, the last fork could be ignored. Soon, a new method for 4 forks was developed. Since, out of the first 3 forks, at least 2 forks were facing the same direction, people trying to replicate Jonah's magic could ensure that the first 3 forks were all facing the same direction. Then, if a student flips one of the first 3, it will be obvious to the guesser, and if they flip the fourth fork, the guesser will see that the first three forks are all facing the same direction and guess that the fourth fork had been flipped. Then, the question of 8 forks remained. How could we, confused students, replicate the telepathic powers of our mentally enlightened instructors? Jonah spilled another hint that would help us achieve similar results to his - for 4 forks, the first three forks in binary could be matched to vertices of a cube, with adjacent vertices one flip away from each other and no vertex adjacent to both 000 and 111. How could we apply this to the 8-fork system? We now knew that for 8 forks, each of the 7 important forks had to match to something, so each vertex had to have 7 connected vertices. Someone suggested a 7-dimensional object - *what???* How does that even work? The question still remains, and until it is answered, Alice and Jonah continue to be either telepathic wizards or transcended aliens from a 7-dimensional universe.

Tuesday's Daily Gather with YouTube

By Alice

This week, for daily gather, we watched a few videos. The first one was about a triangle named Wind. It's a small world, and very flat. Between Wind and her only neighbor, Mr. Ug, are the following: a big rock, a pine log, and a beet peel. Unfortunately, Wind has never met her neighbor, but they do leave notes for each other. We soon realize Wind lives alone on a Möbius strip. After a tragic earthquake rips her world in two, the instructors revealed to us how Wind's relationship with Mr. Ug after the latter swore at her.

We then watched a fractal modeling animation from 1990. This classic began with simulation of Sierpinski's tetrahedron, and ended with multiple rainbow broccoli swirling around on screen, a visualization of two parameter family of 3D extensions to the twindragon.

The next video was about Borromean rings. This topic is about three linked circles, which can not be separated. However, with the removal of any one of the three rings, the other two can easily be separated. I guess it's true what they say, "United we stand, divided we fall." Interestingly enough, these intersecting rings are the logo of the International Mathematical Union (IMU).

The fourth video we watched was also about linked objects as we talked about knots. In math, knots are like normal knots, but the ends are not loose. Because of this, you can not undo a closed loop. We also learned about prime knots and composite knots. While you may know what prime and composite mean for integers, their definitions when referring to knots are not the same. A prime knot is one that can not be separated into smaller knots, while a composite knot is the opposite.

We also watched a video about stereographic projection. This phenomenal phenomenon occurs when shining a focused light through a holey sphere. For example, the first one had a couple of round 4-sided shapes. However, the shadows from the light revealed a perfect square grid. A similar thing occurred with a hexagonal shape. The strange curvy hexagon shapes became a perfect grid of right hexagons.

Some other videos we watched were one called Peace for Triple Piano, an explanation of why $(a+b)^2 = a^2 + 2ab + b^2$ and why $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Unfortunately, the video cut out before doing a 4-Dimensional demonstration. We then watched a short clip from Futurama, and we learn where a certain instructor got the idea for his ID swap daily gather. We concluded the day by rewatching a video from functions. This video, according to MathILy-Er student Julia, was “Fantastic!”. Overall, this movie experience brought us all closer together as a group, and that’s what matters.

Wednesday’s Daily Gather: Almost Right with Aaron

By Aaron

This Wednesday we had a daily gather by Aaron Mesa on right triangles. Wait, no, that is wrong, but I was almost right. This Wednesday, Aaron Fenyes led our daily gather on almost right triangles. We first tried to find isosceles right triangles with integer side lengths, before concluding that it was impossible. We then relaxed the Pythagorean theorem a little bit so a2=b2+c2+1 or a2=b2+c2-1. This gives us triangles with angles very close to right. With these relaxed equations, looked for isosceles almost right triangles, as well as almost right triangles with sides a, b, 2b; 2b, b, a; and a, 3b, 4b. We set to work squaring numbers. There were not a lot of easy to find examples, however, so we created programs on Sage to find examples for us. We also attempted to find equations to generate the next terms in our sequences. We think we can find the next term in the a, b, b, triangle, for instance, with the equation b1+a1=b2 and 2b1+a1=a2. We tried, but failed, to prove our equations. At the end of the session, we learned that by writing the equations a products of matrices instead of two different equations would allow cool patterns to come to light. In addition, they might reveal a lurking communist turkey hidden behind the equations and allow us to find and prove equations for triangles where it is harder to find examples by guess and check, such as Davis’s triangle (a, 3b, 4b), whose first couple values for (a, b) are (8,3), (127, 48), (2024, 765), and (32257, 12192). This daily gather only scratched the surface of almost right triangles; there are many more patterns waiting to be explored and proved.

Thursday’s Daily Gather: Tangle Time with Cathy

By Sophie

Thursday’s Daily Gather started off with some advanced shoelace tying, courtesy of our very own Dr. Catherine Hsu. In the challenge she posed, four students were given a red and a yellow rope, so that each student held onto the end of one rope. Only two operations were allowed for rope-tangling: rotating and twisting. Each rotation moved the position of the people holding the rope 90 degrees, while each twist added one more twist to the knot formed by the two ropes. One of the objectives of the challenge was to ascertain a detangling algorithm for any sequence of twists and rotations that produced a tangle, which we gave a measure called a "tangle number." The tangle number of each sequence was determined by how many rotations and twists the ropes had. If 0 was assumed to be the starting position of the rope, then every twist of the two ropes added one to the tangle number, which we expressed in the relation $T(x) = x + 1$. Eventually, through casework, it was determined that the rotation function was $R(x) =-\frac{1}{x}$. After this fact was determined, Boris proved a detangling algorithm: for a tangle number n, if $n>0$, rotate until $n<0$; if $n<0$, turn until $n>0$ and repeat until 0 is reached. In addition, a tangling algorithm can also be created with the information about $T(x)$ and $R(x)$. To create a tangle with tangle number n, start at a rope with a tangle number $-n$. Finally, apply Boris' algorithm until n is reached.

Friday's Daily Gather: The Schmo and the Shlub with Tom

By Lola

One day, Schmo and Shlub were walking together. All of a sudden, an object appeared in front of Schmo. "A," he said to Shlub to which Schmo replied, "AH." Shlub and Schmo continued laughing by replacing every A with an AH and every H with an A. Every time either of them said "A," they both exhaled and took a step to the right and every time either uttered "H," they both climbed up a step air and inhaled. Schmo and Shlub, wrapped up in their own form of humor, continued replying to each other in this manner, laughing for longer and longer each time. Both Schmo and Shlub have bad lungs and can get hypoxic if they exhale for too long. However, S&S were not afraid. In their laughs, perhaps S&S were smarter than we thought they were. We realized that there was rightfully no need for them to worry because there was no chance of them getting hypoxia. There would never be an infinite sequence of As. We also found that the number of total letters in the series and the number of As and Hs corresponded to the fibonacci sequence. As they climbed higher and higher, we noticed the slope of their steps was 1/(the golden ratio) and their steps could be reconfigured to draw complicated, yet beautiful, fractal sequences. As Schmo and Shlub continued laughing, we realized that their seemingly limited game actually unlocked an amazing rabbithole into the depths of mathematics. Ahahahhahahahhah.
